Chapter 6

Motion in Two Dimensions

What You'll Learn

- You will use Newton's laws and your knowledge of vectors to analyze motion in two dimensions.
- You will solve problems dealing with projectile and circular motion.
- You will solve relative-velocity problems.

Why It's Important

Almost all types of transportation and amusement-park attractions contain at least one element of projectile or circular motion or are affected by relative velocities.

Swinging Around
Before this ride starts to move, the seats hang straight down from their supports. When the ride speeds up, the seats swing out at an angle.

Think About This ▶

When the swings are moving around the circle at a constant speed, are they accelerating?

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LAUNCH Lab How can the motion of a projectile be described?

Question

Can you describe a projectile's motion in both the horizontal and the vertical directions?

Procedure

1. With a marked grid in the background, videotape a ball that is launched with an initial velocity only in the horizontal direction.
2. **Make and Use Graphs** On a sheet of graph paper, draw the location of the ball every 0.1 s (3 frames).
3. Draw two motion diagrams: one for the ball's horizontal motion and one for its vertical motion.

Analysis

How does the vertical motion change as time passes? Does it increase, decrease, or stay the same? How does the horizontal motion change as time passes? Does it increase, decrease, or stay the same?

Critical Thinking Describe the motion of a horizontally launched projectile.

6.1 Projectile Motion

If you observed the movement of a golf ball being hit from a tee, a frog hopping, or a free throw being shot with a basketball, you would notice that all of these objects move through the air along similar paths, as do baseballs, arrows, and bullets. Each path is a curve that moves upward for a distance, and then, after a time, turns and moves downward for some distance. You may be familiar with this curve, called a parabola, from math class.

An object shot through the air is called a **projectile.** A projectile can be a football, a bullet, or a drop of water. After a projectile is launched, what forces are exerted on the projectile? You can draw a free-body diagram of a launched projectile and identify all the forces that are acting on it. No matter what the object is, after a projectile has been given an initial thrust, if you ignore air resistance, it moves through the air only under the force of gravity. The force of gravity is what causes the object to curve downward in a parabolic flight path. Its path through space is called its **trajectory.** If you know the force of the initial thrust on a projectile, you can calculate its trajectory.

▶ Objectives

- **Recognize** that the vertical and horizontal motions of a projectile are independent.
- **Relate** the height, time in the air, and initial vertical velocity of a projectile using its vertical motion, and then determine the range using the horizontal motion.
- **Explain** how the trajectory of a projectile depends upon the frame of reference from which it is observed.

▶ Vocabulary

projectile
trajectory

Horizons Companies

Independence of Motion in Two Dimensions

Think about two softball players warming up for a game, tossing a ball back and forth. What does the path of the ball through the air look like? It looks like a parabola, as you just learned. Imagine that you are standing directly behind one of the players and you are watching the softball as it is being tossed. What would the motion of the ball look like? You would see it go up and back down, just like any object that is tossed straight up in the air. If you were watching the softball from a hot-air balloon high above the field, what motion would you see then? You would see the ball move from one player to the other at a constant speed, just like any object that is given an initial horizontal velocity, such as a hockey puck sliding across ice. The motion of projectiles is a combination of these two motions.

Why do projectiles behave in this way? After a softball leaves a player's hand, what forces are exerted on the ball? If you ignore air resistance, there are no contact forces on the ball. There is only the field force of gravity in the downward direction. How does this affect the ball's motion? Gravity causes the ball to have a downward acceleration.

Figure 6-1 shows the trajectories of two softballs. One was dropped and the other was given an initial horizontal velocity of 2.0 m/s. What is similar about the two paths? Look at their vertical positions. During each flash from the strobe light, the heights of the two softballs are the same. Because the change in vertical position is the same for both, their average vertical velocities during each interval are also the same. The increasingly large distance traveled vertically by the softballs, from one time interval to the next, shows that they are accelerated downward due to the force of gravity. Notice that the horizontal motion of the launched ball does not affect its vertical motion. A projectile launched horizontally has no initial vertical velocity. Therefore, its vertical motion is like that of an object dropped from rest. The downward velocity increases regularly because of the acceleration due to gravity.

MINI LAB

Over the Edge

Obtain two balls, one twice the mass of the other.

1. Predict which ball will hit the floor first when you roll them over the surface of a table and let them roll off the edge.

2. Predict which ball will hit the floor furthest from the table.

3. Explain your predictions.

4. Test your predictions.

Analyze and Conclude

5. Does the mass of the ball affect its motion? Is mass a factor in any of the equations for projectile motion?

Concepts In Motion

Interactive Figure To see an animation on independence of motion in two directions, visit physicspp.com.

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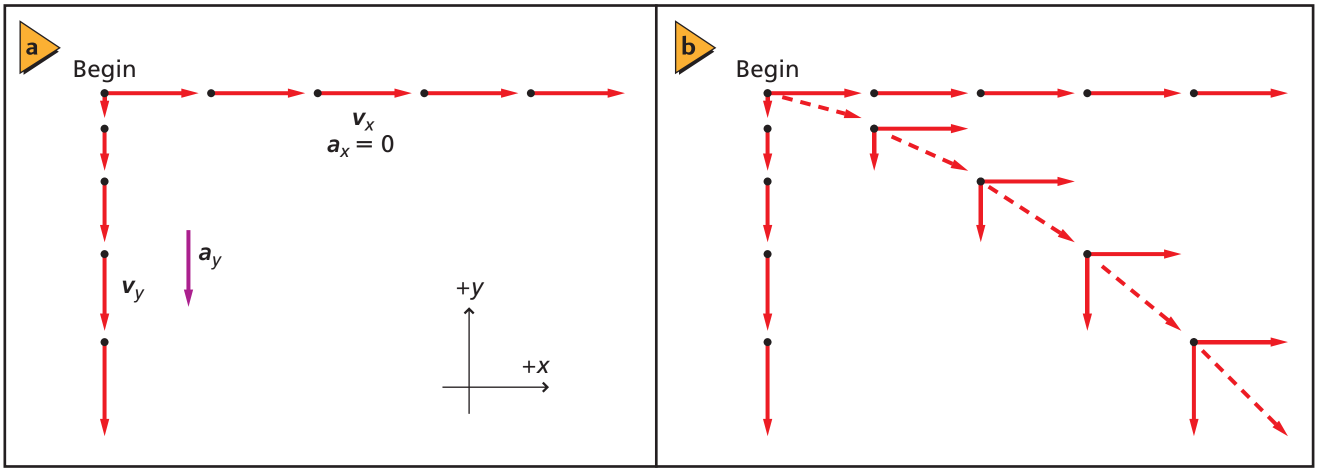
■ **Figure 6-1** The ball on the right was given an initial horizontal velocity. The ball on the left was dropped at the same time from rest. Note that the vertical positions of the two objects are the same during each flash.



■ **Figure 6-2** A object's motion can be broken into its x- and y-components **(a).** When the horizontal and vertical components of the ball's velocity are combined **(b),** the resultant vectors are tangent to a parabola.

Separate motion diagrams for the horizontal and vertical motions are shown in **Figure 6-2a.** The vertical-motion diagram represents the motion of the dropped ball. The horizontal-motion diagram shows the constant velocity in the x-direction of the launched ball. This constant velocity in the horizontal direction is exactly what should be expected because there is no horizontal force acting on the ball.

In **Figure 6-2b,** the horizontal and vertical components are added to form the total velocity vector for the projectile. You can see how the combination of constant horizontal velocity and uniform vertical acceleration produces a trajectory that has a parabolic shape.

PROBLEM-SOLVING **Strategies**

Motion in Two Dimensions

Projectile motion in two dimensions can be determined by breaking the problem into two connected one-dimensional problems.

1. Divide the projectile motion into a vertical motion problem and a horizontal motion problem.
2. The vertical motion of a projectile is exactly that of an object dropped or thrown straight up or straight down. A gravitational force acts on the object and accelerates it by an amount, g. Review Section 3.3 on free fall to refresh your problem-solving skills for vertical motion.
3. Analyzing the horizontal motion of a projectile is the same as solving a constant velocity problem. No horizontal force acts on a projectile when drag due to air resistance is neglected. Consequently, there are no forces acting in the horizontal direction and therefore, no horizontal acceleration; $a_x = 0.0$ m/s. To solve, use the same methods that you learned in Section 2.4.
4. Vertical motion and horizontal motion are connected through the variable of time. The time from the launch of the projectile to the time it hits the target is the same for both vertical motion and horizontal motion. Therefore, solving for time in one of the dimensions, vertical or horizontal, automatically gives you time for the other dimension.

PRACTICE Problems

- Additional Problems, Appendix B
- Solutions to Selected Problems, Appendix C

1. A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.

a. How long does it take the stone to reach the bottom of the cliff?

b. How far from the base of the cliff does the stone hit the ground?

c. What are the horizontal and vertical components of the stone's velocity just before it hits the ground?

2. Lucy and her friend are working at an assembly plant making wooden toy giraffes. At the end of the line, the giraffes go horizontally off the edge of the conveyor belt and fall into a box below. If the box is 0.6 m below the level of the conveyor belt and 0.4 m away from it, what must be the horizontal velocity of giraffes as they leave the conveyor belt?

3. You are visiting a friend from elementary school who now lives in a small town. One local amusement is the ice-cream parlor, where Stan, the short-order cook, slides his completed ice-cream sundaes down the counter at a constant speed of 2.0 m/s to the servers. (The counter is kept very well polished for this purpose.) If the servers catch the sundaes 7.0 cm from the edge of the counter, how far do they fall from the edge of the counter to the point at which the servers catch them?

Projectiles Launched at an Angle

When a projectile is launched at an angle, the initial velocity has a vertical component, as well as a horizontal component. If the object is launched upward, like a ball tossed straight up in the air, it rises with slowing speed, reaches the top of its path, and descends with increasing speed. **Figure 6-3a** shows the separate vertical- and horizontal-motion diagrams for the trajectory. In the coordinate system, the positive *x*-axis is horizontal and the positive *y*-axis is vertical. Note the symmetry. At each point in the vertical direction, the velocity of the object as it is moving upward has the same magnitude as when it is moving downward. The only difference is that the directions of the two velocities are opposite.

Figure 6-3b defines two quantities associated with the trajectory. One is the maximum height, which is the height of the projectile when the vertical velocity is zero and the projectile has only its horizontal-velocity component. The other quantity depicted is the range, *R*, which is the horizontal distance that the projectile travels. Not shown is the flight time, which is how much time the projectile is in the air. For football punts, flight time often is called hang time.

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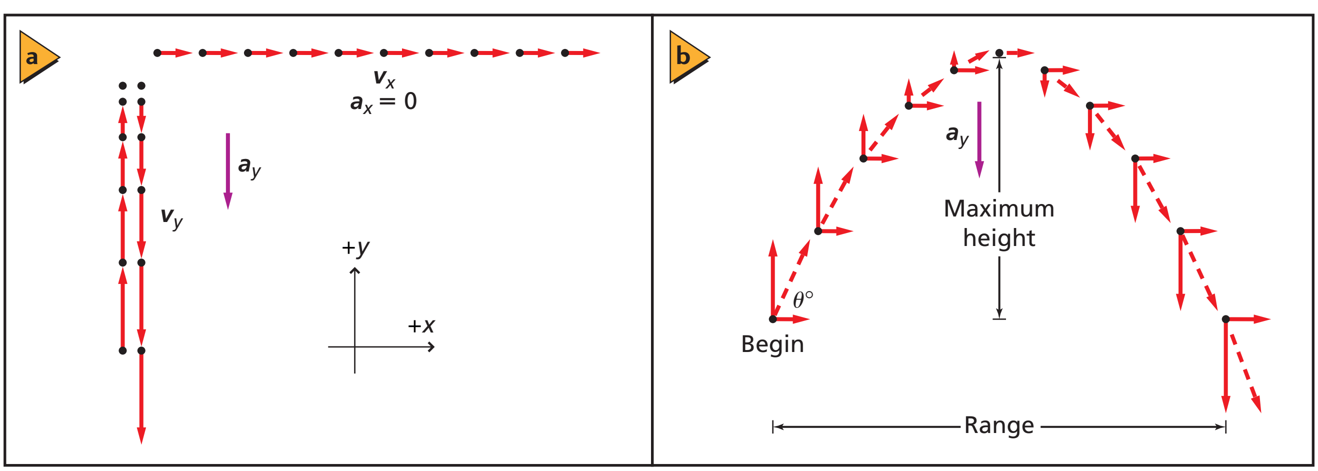
■ **Figure 6-3** The vector sum of $\boldsymbol{v}_x$ and $\boldsymbol{v}_y$ at each position points in the direction of the flight.

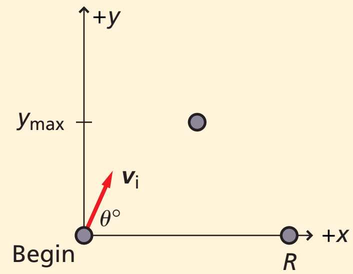


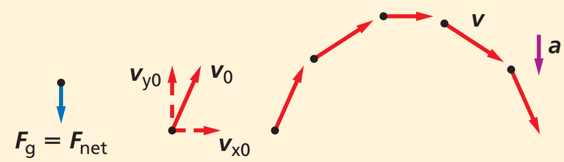
▶ EXAMPLE Problem 1

The Flight of a Ball A ball is launched at 4.5 m/s at 66° above the horizontal. What are the maximum height and flight time of the ball?

1 Analyze and Sketch the Problem

- Establish a coordinate system with the initial position of the ball at the origin.
- Show the positions of the ball at the beginning, at the maximum height, and at the end of the flight.
- Draw a motion diagram showing ***v*, *a*,** and $\boldsymbol{F}_{net}$.





Known:		Unknown:
$y_i = 0.0$ m	$\theta_i = 66°$	$y_{max} = ?$
$v_i = 4.5$ m/s	$a_y = -g$	$t = ?$

2 Solve for the Unknown

Find the y-component of v_i.

$$v_{yi} = v_i(\sin\theta_i)$$
$$= (4.5 \text{ m/s})(\sin 66°) \quad \text{Substitute } v_i = 4.5 \text{ m/s}, \theta_i = 66°$$
$$= 4.1 \text{ m/s}$$

Find an expression for time.

$$v_y = v_{yi} + a_y t$$
$$= v_{yi} - gt \quad \text{Substitute } a_y = -g$$
$$t = \frac{v_{yi} - v_y}{g} \quad \text{Solve for } t.$$

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Personal Tutor For an animation on the flight of a ball, visit physicspp.com.

Solve for the maximum height.

$$y_{max} = y_i + v_{yi}t + \frac{1}{2}at^2$$
$$= y_i + v_{yi}\left(\frac{v_{yi} - v_y}{g}\right) + \frac{1}{2}(-g)\left(\frac{v_{yi} - v_y}{g}\right)^2 \quad \text{Substitute } t = \frac{v_{yi} - v_y}{g}, a = -g$$
$$= 0.0 \text{ m} + (4.1 \text{ m/s})\left(\frac{4.1 \text{ m/s} - 0.0 \text{ m/s}}{9.80 \text{ m/s}^2}\right) + \frac{1}{2}(-9.80 \text{ m/s}^2)\left(\frac{4.1 \text{ m/s} - 0.0 \text{ m/s}}{9.80 \text{ m/s}^2}\right)^2$$
$$= 0.86 \text{ m}$$

Substitute $y_i = 0.0$ m, $v_{yi} = 4.1$ m/s, $v_y = 0.0$ m/s at y_{max}, $g = 9.80$ m/s^2

Solve for the time to return to the launching height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2$$
$$0.0 \text{ m} = 0.0 \text{ m} + v_{yi}t - \frac{1}{2}gt^2 \quad \text{Substitute } y_f = 0.0 \text{ m}, y_i = 0.0 \text{ m}, a = -g$$
$$t = \frac{-v_{yi} \pm \sqrt{v_{yi}^2 - 4\left(-\frac{1}{2}g\right)(0.0 \text{ m})}}{2\left(-\frac{1}{2}g\right)} \quad \text{Use the quadratic formula to solve for } t.$$
$$= \frac{-v_{yi} \pm v_{yi}}{-g}$$
$$= \frac{2v_{yi}}{g} \quad \text{0 is the time the ball left the launch, so use this solution.}$$
$$= \frac{(2)(4.1 \text{ m/s})}{(9.80 \text{ m/s}^2)} \quad \text{Substitute } v_{yi} = 4.1 \text{ m/s}, g = 9.80 \text{ m/s}^2$$
$$= 0.84 \text{ s}$$

3 Evaluate the Answer

- **Are the units correct?** Dimensional analysis verifies that the units are correct.
- **Do the signs make sense?** All should be positive.
- **Are the magnitudes realistic?** 0.84 s is fast, but an initial velocity of 4.5 m/s makes this time reasonable.

PRACTICE Problems

• Additional Problems, Appendix B
• Solutions to Selected Problems, Appendix C

4. A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal, as shown in **Figure 6-4.** Find each of the following. Assume that air resistance is negligible.

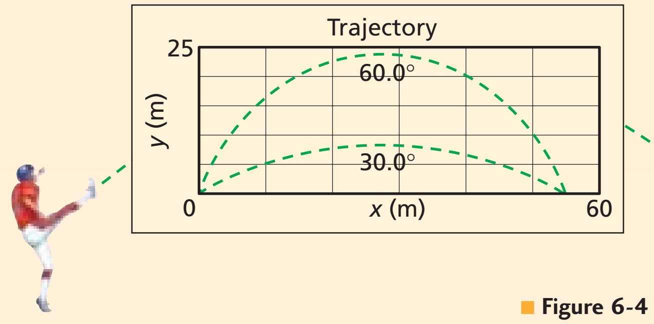
a. the ball's hang time

b. the ball's maximum height

c. the ball's range

5. The player in problem 4 then kicks the ball with the same speed, but at 60.0° from the horizontal. What is the ball's hang time, range, and maximum height?

6. A rock is thrown from a 50.0-m-high cliff with an initial velocity of 7.0 m/s at an angle of 53.0° above the horizontal. Find the velocity vector for when it hits the ground below.



■ **Figure 6-4**

Trajectories Depend upon the Viewer

Suppose you toss a ball up and catch it while riding in a bus. To you, the ball would seem to go straight up and straight down. But what would an observer on the sidewalk see? The observer would see the ball leave your hand, rise up, and return to your hand, but because the bus would be moving, your hand also would be moving. The bus, your hand, and the ball would all have the same horizontal velocity. Thus, the trajectory of the ball would be similar to that of the ball in Example Problem 1.

Air resistance So far, air resistance has been ignored in the analysis of projectile motion. While the effects of air resistance are very small for some projectiles, for others, the effects are large and complex. For example, dimples on a golf ball reduce air resistance and maximize its range. In baseball, the spin of the ball creates forces that can deflect the ball. For now, just remember that the force due to air resistance does exist and it can be important.

6.1 Section Review

7. Projectile Motion Two baseballs are pitched horizontally from the same height, but at different speeds. The faster ball crosses home plate within the strike zone, but the slower ball is below the batter's knees. Why does the faster ball not fall as far as the slower one?

8. Free-Body Diagram An ice cube slides without friction across a table at a constant velocity. It slides off the table and lands on the floor. Draw free-body and motion diagrams of the ice cube at two points on the table and at two points in the air.

9. Projectile Motion A softball is tossed into the air at an angle of 50.0° with the vertical at an initial velocity of 11.0 m/s. What is its maximum height?

10. Projectile Motion A tennis ball is thrown out a window 28 m above the ground at an initial velocity of 15.0 m/s and 20.0° below the horizontal. How far does the ball move horizontally before it hits the ground?

11. Critical Thinking Suppose that an object is thrown with the same initial velocity and direction on Earth and on the Moon, where g is one-sixth that on Earth. How will the following quantities change?

a. v_x

b. the object's time of flight

c. y_{max}

d. R

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6.2 Circular Motion

Consider an object moving in a circle at a constant speed, such as a stone being whirled on the end of a string or a fixed horse on a merry-go-round. Are these objects accelerating? At first, you might think that they are not because their speeds do not change. However, remember that acceleration is the change in velocity, not just the change in speed. Because their direction is changing, the objects must be accelerating.

▶ **Objectives**

- **Explain** why an object moving in a circle at a constant speed is accelerated.
- **Describe** how centripetal acceleration depends upon the object's speed and the radius of the circle.
- **Identify** the force that causes centripetal acceleration.

▶ **Vocabulary**

uniform circular motion
centripetal acceleration
centripetal force

Describing Circular Motion

Uniform circular motion is the movement of an object or particle trajectory at a constant speed around a circle with a fixed radius. The position of an object in uniform circular motion, relative to the center of the circle, is given by the position vector $\boldsymbol{r}$, shown in **Figure 6-5a.** As the object moves around the circle, the length of the position vector does not change, but its direction does. To find the object's velocity, you need to find its displacement vector over a time interval. The change in position, or the object's displacement, is represented by $\Delta\boldsymbol{r}$. **Figure 6-5b** shows two position vectors: $\boldsymbol{r}_1$ at the beginning of a time interval, and $\boldsymbol{r}_2$ at the end of the time interval. Remember that a position vector is a displacement vector with its tail at the origin. In the vector diagram, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are subtracted to give the resultant $\Delta\boldsymbol{r}$, the displacement during the time interval. You know that a moving object's average velocity is $\Delta\boldsymbol{d}/\Delta t$, so for an object in circular motion, $\bar{\boldsymbol{v}} = \Delta\boldsymbol{r}/\Delta t$. The velocity vector has the same direction as the displacement, but a different length. You can see in **Figure 6-6a** that the velocity is at right angles to the position vector, which is tangent to its circular path. As the velocity vector moves around the circle, its direction changes but its length remains the same.

What is the direction of the object's acceleration? Figure 6-6a shows the velocity vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ at the beginning and end of a time interval. The difference in the two vectors, $\Delta\boldsymbol{v}$, is found by subtracting the vectors, as shown in **Figure 6-6b.** The average acceleration, $\bar{\boldsymbol{a}} = \Delta\boldsymbol{v}/\Delta t$, is in the same direction as $\Delta\boldsymbol{v}$; that is, toward the center of the circle. Repeat this process for several other time intervals when the object is in different locations on the circle. As the object moves around the circle, the direction of the acceleration vector changes, but its length remains the same. Notice that the acceleration vector of an object in uniform circular motion always points in toward the center of the circle. For this reason, the acceleration of such an object is called center-seeking or **centripetal acceleration.**

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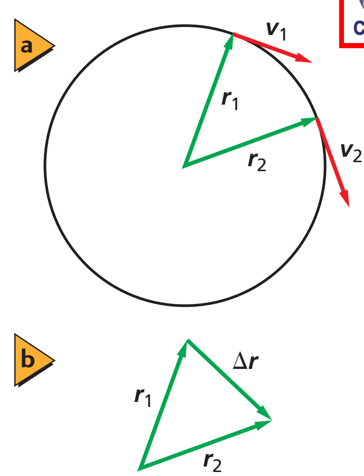
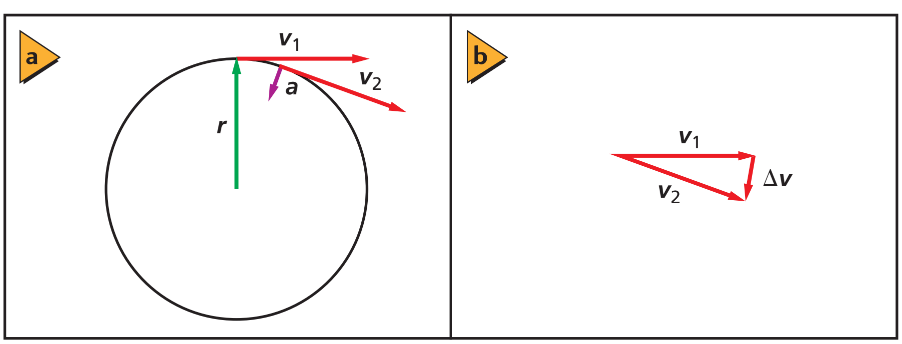


■ **Figure 6-5** The displacement, Δr, of an object in circular motion, divided by the time interval in which the displacement occurs, is the object's average velocity during that time interval.



■ **Figure 6-6** The direction of the change in velocity is toward the center of the circle, and so the acceleration vector also points to the center of the circle.

Centripetal Acceleration

What is the magnitude of an object's centripetal acceleration? Compare the triangle made from the position vectors in Figure 6-5b with the triangle made by the velocity vectors in Figure 6-6b. The angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ is the same as that between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Therefore, the two triangles formed by subtracting the two sets of vectors are similar triangles, and the ratios of the lengths of two corresponding sides are equal. Thus, $\Delta r/r = \Delta v/v$. The equation is not changed if both sides are divided by Δt.

$$\frac{\Delta r}{r\Delta t} = \frac{\Delta v}{v\Delta t}$$

However, $v = \Delta r/\Delta t$ and $a = \Delta v/\Delta t$.

$$\frac{1}{r}\left(\frac{\Delta r}{\Delta t}\right) = \frac{1}{v}\left(\frac{\Delta v}{\Delta t}\right)$$

Substituting $v = \Delta r/\Delta t$ in the left-hand side and $a = \Delta v/\Delta t$ in the right-hand side gives the following equation.

$$\frac{v}{r} = \frac{a}{v}$$

Solve this equation for acceleration and give it the special symbol a_c, for centripetal acceleration.

Centripetal Acceleration $a_c = \frac{v^2}{r}$

Centripetal acceleration always points to the center of the circle. Its magnitude is equal to the square of the speed, divided by the radius of motion.

How can you measure the speed of an object moving in a circle? One way is to measure its period, T, the time needed for the object to make one complete revolution. During this time, the object travels a distance equal to the circumference of the circle, $2\pi r$. The object's speed, then, is represented by $v = 2\pi r/T$. If this expression is substituted for v in the equation for centripetal acceleration, the following equation is obtained.

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Because the acceleration of an object moving in a circle is always in the direction of the net force acting on it, there must be a net force toward the center of the circle. This force can be provided by any number of agents. For Earth circling the Sun, the force is the Sun's gravitational force on Earth, as you'll learn in Chapter 7. When a hammer thrower swings the hammer, as in **Figure 6-7,** the force is the tension in the chain attached to the massive ball. When an object moves in a circle, the net force toward the center of the circle is called the **centripetal force.** To accurately analyze centripetal acceleration situations, you must identify the agent of the force that causes the acceleration. Then you can apply Newton's second law for the component in the direction of the acceleration in the following way.

Newton's Second Law for Circular Motion $F_{net} = ma_c$

The net centripetal force on an object moving in a circle is equal to the object's mass, times the centripetal acceleration.

APPLYING PHYSICS

▶ **Space Elevators** Scientists are considering the use of space elevators as a low-cost transportation system to space. A cable would be anchored to a station at Earth's equator, and the cable would extend almost 35,800 km from Earth's surface. The cable would be attached to a counterweight and would stay extended due to centripetal force. Special magnetically powered vehicles would then travel along the cable. ◀

■ **Figure 6-7** When the thrower lets go, the hammer initially moves in a straight line that is tangent to the point of release. Then it follows a trajectory like that of any object released into the air with an initial horizontal velocity.

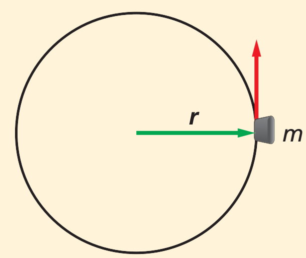
Denis Charlet/AFP/Getty Images

When solving problems, you have found it useful to choose a coordinate system with one axis in the direction of the acceleration. For circular motion, the direction of the acceleration is always toward the center of the circle. Rather than labeling this axis *x* or *y*, call it *c*, for centripetal acceleration. The other axis is in the direction of the velocity, tangent to the circle. It is labeled *tang* for tangential. You will apply Newton's second law in these directions, just as you did in the two-dimensional problems in Chapter 5. Remember that centripetal force is just another name for the net force in the centripetal direction. It is the sum of all the real forces, those for which you can identify agents that act along the centripetal axis.

In the case of the hammer thrower in Figure 6-7, in what direction does the hammer fly when the chain is released? Once the contact force of the chain is gone, there is no force accelerating the hammer toward the center of the circle, so the hammer flies off in the direction of its velocity, which is tangent to the circle. Remember, if you cannot identify the agent of the force, then it does not exist.

▶ EXAMPLE Problem 2

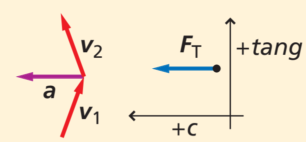
Uniform Circular Motion **A 13-g rubber stopper is attached to a 0.93-m string. The stopper is swung in a horizontal circle, making one revolution in 1.18 s. Find the tension force exerted by the string on the stopper.**



1 Analyze and Sketch the Problem

- Draw a free-body diagram for the swinging stopper.
- Include the radius and the direction of motion.
- Establish a coordinate system labeled *tang* and *c*. The directions of *a* and F_T are parallel to *c*.

Known:	**Unknown:**
$m = 13$ g	$F_T = ?$
$r = 0.93$ m	
$T = 1.18$ s	



2 Solve for the Unknown

Find the centripetal acceleration.

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(0.93\ \text{m})}{(1.18\ \text{s})^2} \quad \text{Substitute } r = 0.93\ \text{m},\ T = 1.18\ \text{s}$$

$$= 26\ \text{m/s}^2$$

Use Newton's second law to find the tension in the string.

$$F_T = ma_c$$

$$= (0.013\ \text{kg})(26\ \text{m/s}^2) \quad \text{Substitute } m = 0.013\ \text{kg},\ a_c = 26\ \text{m/s}^2$$

$$= 0.34\ \text{N}$$

Math Handbook
Operations with Significant Digits
pages 835–836

3 Evaluate the Answer

- **Are the units correct?** Dimensional analysis verifies that *a* is in m/s^2 and *F* is in N.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The force is almost three times the weight of the stopper, and the acceleration is almost three times that of gravity, which is reasonable for such a light object.

PRACTICE Problems

- Additional Problems, Appendix B
- Solutions to Selected Problems, Appendix C

12. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m. What is the centripetal acceleration of the runner, and what agent exerts force on the runner?

13. A car racing on a flat track travels at 22 m/s around a curve with a 56-m radius. Find the car's centripetal acceleration. What minimum coefficient of static friction between the tires and road is necessary for the car to round the curve without slipping?

14. An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in km) that the pilot can make and keep the centripetal acceleration under 5.0 m/s^2?

15. A 45-kg merry-go-round worker stands on the ride's platform 6.3 m from the center. If her speed as she goes around the circle is 4.1 m/s, what is the force of friction necessary to keep her from falling off the platform?

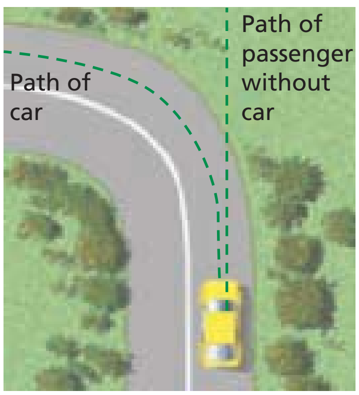


Figure 6-8 The passenger would move forward in a straight line if the car did not exert an inward force.

A Nonexistent Force

If a car makes a sharp left turn, a passenger on the right side might be thrown against the right door. Is there an outward force on the passenger? Consider a similar situation. If a car in which you are riding stops suddenly, you will be thrown forward into your safety belt. Is there a forward force on you? No, because according to Newton's first law, you will continue moving with the same velocity unless there is a net force acting on you. The safety belt applies the force that accelerates you to a stop. **Figure 6-8** shows a car turning to the left as viewed from above. A passenger in the car would continue to move straight ahead if it were not for the force of the door acting in the direction of the acceleration; that is, toward the center of the circle. Thus, there is no outward force on the passenger. The so-called centrifugal, or outward force, is a fictitious, nonexistent force. Newton's laws are able to explain motion in both straight lines and circles.

6.2 Section Review

16. Uniform Circular Motion What is the direction of the force that acts on the clothes in the spin cycle of a washing machine? What exerts the force?

17. Free-Body Diagram You are sitting in the back-seat of a car going around a curve to the right. Sketch motion and free-body diagrams to answer the following questions.

a. What is the direction of your acceleration?

b. What is the direction of the net force that is acting on you?

c. What exerts this force?

18. Centripetal Force If a 40.0-g stone is whirled horizontally on the end of a 0.60-m string at a speed of 2.2 m/s, what is the tension in the string?

19. Centripetal Acceleration A newspaper article states that when turning a corner, a driver must be careful to balance the centripetal and centrifugal forces to keep from skidding. Write a letter to the editor that critiques this article.

20. Centripetal Force A bowling ball has a mass of 7.3 kg. If you move it around a circle with a radius of 0.75 m at a speed of 2.5 m/s, what force would you have to exert on it?

21. Critical Thinking Because of Earth's daily rotation, you always move with uniform circular motion. What is the agent that supplies the force that accelerates you? How does this motion affect your apparent weight?

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6.3 Relative Velocity

Suppose that you are in a school bus that is traveling at a velocity of 8 m/s in a positive direction. You walk with a velocity of 3 m/s toward the front of the bus. If a friend of yours is standing on the side of the road watching the bus with you on it go by, how fast would your friend say that you are moving? If the bus is traveling at 8 m/s, this means that the velocity of the bus is 8 m/s, as measured by your friend in a coordinate system fixed to the road. When you are standing still, your velocity relative to the road is also 8 m/s, but your velocity relative to the bus is zero. Walking at 1 m/s toward the front of the bus means that your velocity is measured relative to the bus. The problem can be rephrased as follows: Given the velocity of the bus relative to the road and your velocity relative to the bus, what is your velocity relative to the road?

A vector representation of this problem is shown in **Figure 6-9a.** After studying it, you will find that your velocity relative to the street is 9 m/s, the sum of 8 m/s and 1 m/s. Suppose that you now walk at the same speed toward the rear of the bus. What would be your velocity relative to the road? **Figure 6-9b** shows that because the two velocities are in opposite directions, the resultant velocity is 7 m/s, the difference between 8 m/s and 1 m/s. You can see that when the velocities are along the same line, simple addition or subtraction can be used to determine the relative velocity.

Take a closer look at how these results were obtained and see if you can find a mathematical rule to describe how velocities are combined in these relative-velocity situations. For the above situation, you can designate the velocity of the bus relative to the road as $\boldsymbol{v}_{b/r}$, your velocity relative to the bus as $\boldsymbol{v}_{y/b}$, and the velocity of you relative to the road as $\boldsymbol{v}_{y/r}$. To find the velocity of you relative to the road in both cases, you vectorially added the velocities of you relative to the bus and the bus relative to the road. Mathematically, this is represented as $\boldsymbol{v}_{y/b} + \boldsymbol{v}_{b/r} = \boldsymbol{v}_{y/r}$. The more general form of this equation is as follows.

Relative Velocity $\boldsymbol{v}_{a/b} + \boldsymbol{v}_{b/c} = \boldsymbol{v}_{a/c}$

The relative velocity of object a to object c is the vector sum of object a's velocity relative to object b and object b's velocity relative to object c.

CHALLENGE PROBLEM

Phillipe whirls a stone of mass *m* on a rope in a perfect horizontal circle above his head such that the stone is at a height, *h*, above the ground. The circle has a radius of *r*, and the tension in the rope is *T*. Suddenly the rope breaks and the stone falls to the ground. The stone travels a horizontal distance, *s*, from the time the rope breaks until it impacts the ground. Find a mathematical expression for *s* in terms of *T*, *r*, *m*, and *h*. Does your expression change if Phillipe is walking 0.50 m/s relative to the ground?

▶ **Objectives**

- **Analyze** situations in which the coordinate system is moving.
- **Solve** relative-velocity problems.

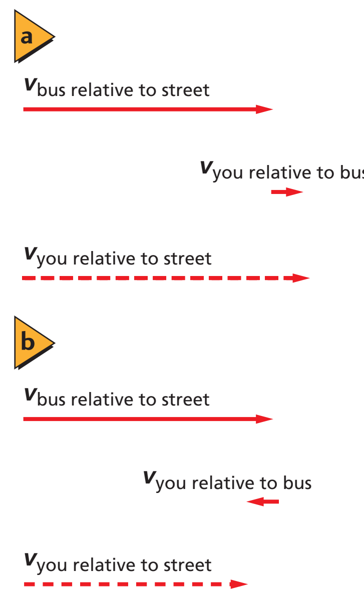


Figure 6-9 When a coordinate system is moving, two velocities are added if both motions are in the same direction and one is subtracted from the other if the motions are in opposite directions.

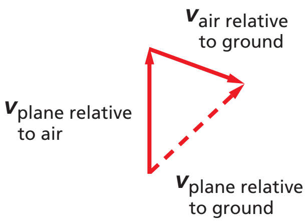


Figure 6-10 The plane's velocity relative to the ground can be obtained by vector addition.

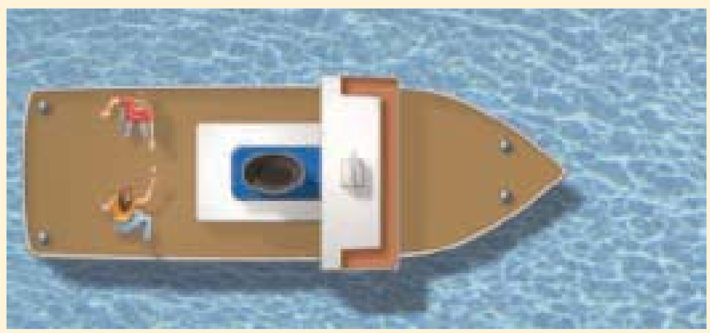
This method for adding relative velocities also applies to motion in two dimensions. For example, airline pilots cannot expect to reach their destinations by simply aiming their planes along a compass direction. They must take into account the plane's speed relative to the air, which is given by their airspeed indicators, and their direction of flight relative to the air. They also must consider the velocity of the wind at the altitude they are flying relative to the ground. These two vectors must be combined, as shown in **Figure 6-10**, to obtain the velocity of the airplane relative to the ground. The resultant vector tells the pilot how fast and in what direction the plane must travel relative to the ground to reach its destination. A similar situation occurs for boats traveling on water with a flowing current.

EXAMPLE Problem 3

Relative Velocity of a Marble Ana and Sandra are riding on a ferry boat that is traveling east at a speed of 4.0 m/s. Sandra rolls a marble with a velocity of 0.75 m/s north, straight across the deck of the boat to Ana. What is the velocity of the marble relative to the water?

1 Analyze and Sketch the Problem

- Establish a coordinate system.
- Draw vectors to represent the velocities of the boat relative to the water and the marble relative to the boat.



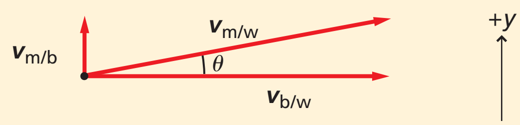
Known:

$v_{b/w} = 4.0$ m/s

$v_{m/b} = 0.75$ m/s

Unknown:

$v_{m/w} = ?$



2 Solve for the Unknown

Because the two velocities are at right angles, use the Pythagorean theorem.

$$v_{m/w}{}^2 = v_{b/w}{}^2 + v_{m/b}{}^2$$

$$v_{m/w} = \sqrt{v_{b/w}{}^2 + v_{m/b}{}^2}$$

$$= \sqrt{(4.0\text{ m/s})^2 + (0.75\text{ m/s})^2}$$

Substitute $v_{b/w} = 4.0$ m/s, $v_{m/b} = 0.75$ m/s

$$= 4.1\text{ m/s}$$

Find the angle of the marble's motion.

$$\theta = \tan^{-1}\left(\frac{v_{m/b}}{v_{b/w}}\right)$$

$$= \tan^{-1}\left(\frac{0.75\text{ m/s}}{4.0\text{ m/s}}\right)$$

Substitute $v_{b/w} = 4.0$ m/s, $v_{m/b} = 0.75$ m/s

$$= 11° \text{ north of east}$$

The marble is traveling 4.1 m/s at 11° north of east.

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Inverses of Sine, Cosine, and Tangent
page 856

3 Evaluate the Answer

- **Are the units correct?** Dimensional analysis verifies that the velocity is in m/s.
- **Do the signs make sense?** The signs should all be positive.
- **Are the magnitudes realistic?** The resulting velocity is of the same order of magnitude as the velocities given in the problem.

PRACTICE Problems

- Additional Problems, Appendix B
- Solutions to Selected Problems, Appendix C

22. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?

23. Rafi is pulling a toy wagon through the neighborhood at a speed of 0.75 m/s. A caterpillar in the wagon is crawling toward the rear of the wagon at a rate of 2.0 cm/s. What is the caterpillar's velocity relative to the ground?

24. A boat is rowed directly upriver at a speed of 2.5 m/s relative to the water. Viewers on the shore see that the boat is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?

25. An airplane flies due north at 150 km/h relative to the air. There is a wind blowing at 75 km/h to the east relative to the ground. What is the plane's speed relative to the ground?

Another example of combined relative velocities is the navigation of migrating neotropical songbirds. In addition to knowing in which direction to fly, a bird must account for its speed relative to the air and its direction relative to the ground. If a bird tries to fly over the Gulf of Mexico into too strong a headwind, it will run out of energy before it reaches the other shore and will perish. Similarly, the bird must account for crosswinds or it will not reach its destination. You can add relative velocities even if they are at arbitrary angles by using the graphical methods that you learned in Chapter 5.

Biology Connection

Remember that the key to properly analyzing a two-dimensional relative-velocity situation is drawing the proper triangle to represent the three velocities. Once you have this triangle, you simply apply your knowledge of vector addition from Chapter 5. If the situation contains two velocities that are perpendicular to each other, you can find the third by applying the Pythagorean theorem; however, if the situation has no right angles, you will need to use one or both of the laws of sines and cosines.

6.3 Section Review

26. Relative Velocity A fishing boat with a maximum speed of 3 m/s relative to the water is in a river that is flowing at 2 m/s. What is the maximum speed the boat can obtain relative to the shore? The minimum speed? Give the direction of the boat, relative to the river's current, for the maximum speed and the minimum speed relative to the shore.

27. Relative Velocity of a Boat A powerboat heads due northwest at 13 m/s relative to the water across a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat relative to the shore?

28. Relative Velocity An airplane flies due south at 175 km/h relative to the air. There is a wind blowing at 85 km/h to the east relative to the ground. What are the plane's speed and direction relative to the ground?

29. A Plane's Relative Velocity An airplane flies due north at 235 km/h relative to the air. There is a wind blowing at 65 km/h to the northeast relative to the ground. What are the plane's speed and direction relative to the ground?

30. Relative Velocity An airplane has a speed of 285 km/h relative to the air. There is a wind blowing at 95 km/h at 30.0° north of east relative to Earth. In which direction should the plane head to land at an airport due north of its present location? What is the plane's speed relative to the ground?

31. Critical Thinking You are piloting a boat across a fast-moving river. You want to reach a pier directly opposite your starting point. Describe how you would navigate the boat in terms of the components of your velocity relative to the water.

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PHYSICS LAB • Design Your Own

On Target

In this activity, you will analyze several factors that affect the motion of a projectile and use your understanding of these factors to predict the path of a projectile. Finally, you will design a projectile launcher and hit a target a known distance away.

QUESTION

What factors affect the path of a projectile?

Objectives

- **Formulate models** and then summarize the factors that affect the motion of a projectile.
- **Use models** to predict where a projectile will land.

Safety Precautions

Possible Materials

duct tape
plastic ware
rubber bands
paper clips
paper
masking tape
wood blocks
nails
hammer
PVC tubing
handsaw
scissors
coat hanger
chicken wire
wire cutter

Procedure

1. Brainstorm and list as many factors as you are able to think of that may affect the path of a projectile.
2. Create a design for your projectile launcher and decide what object will be your projectile shot by your launcher.
3. Taking the design of your launcher into account, determine which two factors are most likely to have a significant effect on the flight path of your projectile.
4. Check the design of your launcher and discuss your two factors with your teacher and make any necessary changes to your setup before continuing.
5. Create a method for determining what effect these two factors will have on the path of your projectile.
6. Have your teacher approve your method before collecting data.

Horizons Companies

Data Table 1

Launch Angle (deg)	Distance Projectile Travels (cm)

Data Table 2

Distance Rubber Band Is Pulled Back (cm)	Distance Projectile Travels (cm)

Analyze

1. **Make and Use Graphs** Make graphs of your data to help you predict how to use your launcher to hit a target.
2. **Analyze** What are the relationships between each variable you have tested and the distance the projectile travels?

Conclude and Apply

1. What were the main factors influencing the path of the projectile?
2. Predict the conditions necessary to hit a target provided by your teacher.
3. **Explain** If you have a perfect plan and still miss the target on your first try, is there a problem with the variability of laws of physics? Explain.
4. Launch your projectile at the target. If you miss, make the necessary adjustments and try again.

Going Further

1. How might your data have varied if you did this experiment outside? Would there be any additional factors affecting the motion of your projectile?
2. How might the results of your experiment be different if the target was elevated above the height of the launcher?
3. How might your experiment differ if the launcher was elevated above the height of the target?

Real-World Physics

1. When a kicker attempts a field goal, do you think it is possible for him to miss because he kicked it too high? Explain.
2. If you wanted to hit a baseball as far as possible, what would be the best angle to hit the ball?

Physics Online

To find out more about projectile motion, visit the Web site: **physicspp.com**

Future Technology

Spinning Space Stations

There is a lot going on aboard the *International Space Station* (ISS). Scientists from different countries are conducting experiments and making observations. They have seen water drops form as floating spheres and have grown peas in space to test whether crops can be grown in weightlessness.

One goal of the ISS is to examine the effects on the human body when living in space for prolonged periods of time. If negative health effects can be identified, perhaps they can be prevented. This could give humans the option of living in space for long periods of time.

Harmful effects of weightlessness have been observed. On Earth, muscles have gravity to push and pull against. Muscles weaken from disuse if this resistance is removed. Bones can weaken for the same reason. Also, blood volume can decrease. On Earth, gravity pulls blood downward so it collects in the lower legs. In weightlessness, the blood can more easily collect in an astronaut's head. The brain senses the extra blood and sends a signal to make less of it.

Long-term life in space is hindered by the practical challenges of weightlessness as well. Imagine how daily life would change. Everything must be strapped or bolted down. You would have to be strapped down to a bed to sleep in one. Your life would be difficult in a space station unless the space station could be modified to simulate gravity. How could this be done?

The Rotating Space Station Have you ever been on a human centrifuge—a type of amusement park ride that uses centripetal force? Everyone stands against the walls of a big cylinder. Then the cylinder begins to rotate faster and faster until the riders are pressed against the walls. Because of the centripetal acceleration, the riders are held there so that even when the floor drops down they are held securely against the walls of the whirling container.

A space station could be designed that uses the effects of centripetal motion as a replacement for gravity. Imagine a space station in the form of a large ring. The space station and all the objects and occupants inside would float weightlessly inside. If the ring were made to spin, unattached objects would be held against the ring's outer edge because of the centripetal motion. If the space station spun at the right rate and if it had the right diameter, the centripetal motion would cause the occupants to experience a force of the same magnitude as gravity. The down direction in the space station would be what an observer outside the station would see as radially outward, away from the ring's center.

This is an artist's rendition of a rotating space station.

Centripetal acceleration is directly proportional to the distance from the center of a rotating object. A rotating space station could be built in the form of concentric rings, each ring experiencing a different gravity. The innermost rings would experience the smallest gravity, while outermost rings would experience the largest force. You could go from floating peacefully in a low-gravity ring to standing securely in the simulated Earth-gravity ring.

Going Further

1. **Research** What factors must engineers take into account in order to make a rotating space station that can simulate Earth's gravity?
2. **Apply** You are an astronaut aboard a rotating space station. You feel pulled by gravity against the floor. Explain what is really going on in terms of Newton's laws and centripetal force.
3. **Critical Thinking** What benefits does a rotating space station offer its occupants? What are the negative features?

Chapter 6 Study Guide

6.1 Projectile Motion

Vocabulary

- projectile (p. 147)
- trajectory (p. 147)

Key Concepts

- The vertical and horizontal motions of a projectile are independent.
- The vertical motion component of a projectile experiences a constant acceleration.
- When there is no air resistance, the horizontal motion component does not experience an acceleration and has constant velocity.
- Projectile problems are solved by first using the vertical motion to relate height, time in the air, and initial vertical velocity. Then the distance traveled horizontally is found.
- The range of a projectile depends upon the acceleration due to gravity and upon both components of the initial velocity.
- The curved flight path that is followed by a projectile is called a parabola.

6.2 Circular Motion

Vocabulary

- uniform circular motion (p. 153)
- centripetal acceleration (p. 153)
- centripetal force (p. 154)

Key Concepts

- An object moving in a circle at a constant speed accelerates toward the center of the circle, and therefore, it has centripetal acceleration.
- Centripetal acceleration depends directly on the square of the object's speed and inversely on the radius of the circle.

$$a_c = \frac{v^2}{r}$$

- The centripetal acceleration for an object traveling in a circle can also be expressed as a function of its period, T.

$$a_c = \frac{4\pi^2 r}{T^2}$$

- A net force must be exerted toward the circle's center to cause centripetal acceleration.

$$F_{net} = ma_c$$

- The velocity vector of an object with a centripetal acceleration is always tangent to the circular path.

6.3 Relative Velocity

Key Concepts

- Vector addition can be used to solve problems involving relative velocities.

$$\boldsymbol{v}_{a/b} + \boldsymbol{v}_{b/c} = \boldsymbol{v}_{a/c}$$

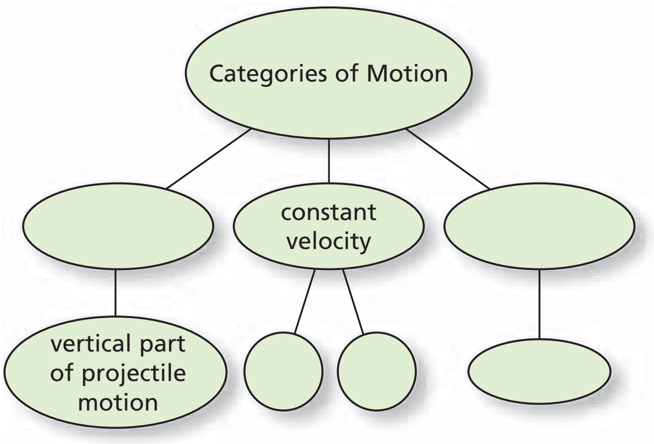
- The key to properly analyzing a two-dimensional relative-velocity problem is drawing the proper triangle to represent all three velocity vectors.

Chapter 6

Assessment

Concept Mapping

32. Use the following terms to complete the concept map below: *constant speed, horizontal part of projectile motion, constant acceleration, relative-velocity motion, uniform circular motion.*



Mastering Concepts

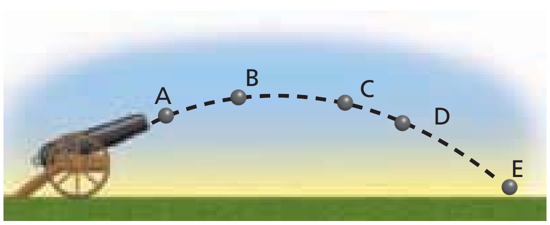
33. Consider the trajectory of the cannonball shown in **Figure 6-11.** (6.1)

a. Where is the magnitude of the vertical-velocity component largest?

b. Where is the magnitude of the horizontal-velocity component largest?

c. Where is the vertical-velocity smallest?

d. Where is the magnitude of the acceleration smallest?



■ **Figure 6-11**

34. A student is playing with a radio-controlled race car on the balcony of a sixth-floor apartment. An accidental turn sends the car through the railing and over the edge of the balcony. Does the time it takes the car to fall depend upon the speed it had when it left the balcony? (6.1)

35. An airplane pilot flying at constant velocity and altitude drops a heavy crate. Ignoring air resistance, where will the plane be relative to the crate when the crate hits the ground? Draw the path of the crate as seen by an observer on the ground. (6.1)

36. Can you go around a curve with the following accelerations? Explain.

a. zero acceleration

b. constant acceleration (6.2)

37. To obtain uniform circular motion, how must the net force depend on the speed of the moving object? (6.2)

38. If you whirl a yo-yo about your head in a horizontal circle, in what direction must a force act on the yo-yo? What exerts the force? (6.2)

39. Why is it that a car traveling in the opposite direction as the car in which you are riding on the freeway often looks like it is moving faster than the speed limit? (6.3)

Applying Concepts

40. Projectile Motion Analyze how horizontal motion can be uniform while vertical motion is accelerated. How will projectile motion be affected when drag due to air resistance is taken into consideration?

41. Baseball A batter hits a pop-up straight up over home plate at an initial velocity of 20 m/s. The ball is caught by the catcher at the same height that it was hit. At what velocity does the ball land in the catcher's mitt? Neglect air resistance.

42. Fastball In baseball, a fastball takes about $\frac{1}{2}$ s to reach the plate. Assuming that such a pitch is thrown horizontally, compare the distance the ball falls in the first $\frac{1}{4}$ s with the distance it falls in the second $\frac{1}{4}$ s.

43. You throw a rock horizontally. In a second horizontal throw, you throw the rock harder and give it even more speed.

a. How will the time it takes the rock to hit the ground be affected? Ignore air resistance.

b. How will the increased speed affect the distance from where the rock left your hand to where the rock hits the ground?

44. Field Biology A zoologist standing on a cliff aims a tranquilizer gun at a monkey hanging from a distant tree branch. The barrel of the gun is horizontal. Just as the zoologist pulls the trigger, the monkey lets go and begins to fall. Will the dart hit the monkey? Ignore air resistance.

45. Football A quarterback throws a football at 24 m/s at a 45° angle. If it takes the ball 3.0 s to reach the top of its path and the ball is caught at the same height at which it is thrown, how long is it in the air? Ignore air resistance.

46. Track and Field You are working on improving your performance in the long jump and believe that the information in this chapter can help. Does the height that you reach make any difference to your jump? What influences the length of your jump?

 For more problems, go to Additional Problems, Appendix B.

47. Imagine that you are sitting in a car tossing a ball straight up into the air.
a. If the car is moving at a constant velocity, will the ball land in front of, behind, or in your hand?
b. If the car rounds a curve at a constant speed, where will the ball land?

48. You swing one yo-yo around your head in a horizontal circle. Then you swing another yo-yo with twice the mass of the first one, but you don't change the length of the string or the period. How do the tensions in the strings differ?

49. Car Racing The curves on a race track are banked to make it easier for cars to go around the curves at high speeds. Draw a free-body diagram of a car on a banked curve. From the motion diagram, find the direction of the acceleration.
a. What exerts the force in the direction of the acceleration?
b. Can you have such a force without friction?

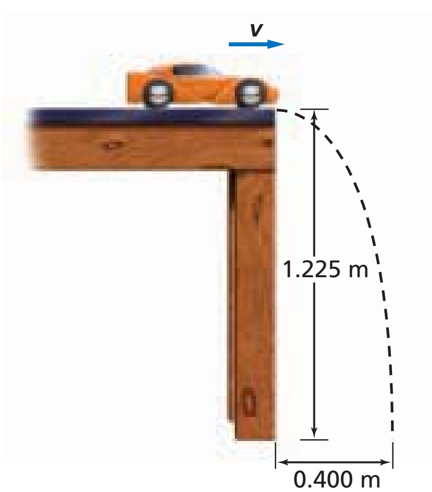
50. Driving on the Highway Explain why it is that when you pass a car going in the same direction as you on the freeway, it takes a longer time than when you pass a car going in the opposite direction.

Mastering Problems

6.1 Projectile Motion

51. You accidentally throw your car keys horizontally at 8.0 m/s from a cliff 64-m high. How far from the base of the cliff should you look for the keys?

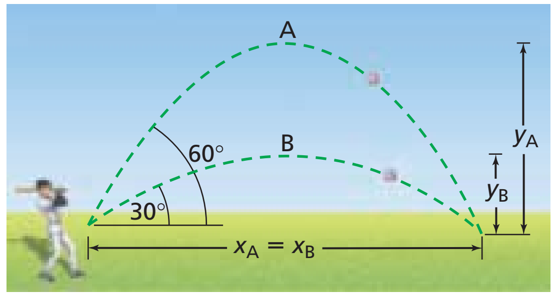
52. The toy car in **Figure 6-12** runs off the edge of a table that is 1.225-m high. The car lands 0.400 m from the base of the table.
a. How long did it take the car to fall?
b. How fast was the car going on the table?



■ **Figure 6-12**

53. A dart player throws a dart horizontally at 12.4 m/s. The dart hits the board 0.32 m below the height from which it was thrown. How far away is the player from the board?

54. The two baseballs in **Figure 6-13** were hit with the same speed, 25 m/s. Draw separate graphs of y versus t and x versus t for each ball.



■ **Figure 6-13**

55. Swimming You took a running leap off a high-diving platform. You were running at 2.8 m/s and hit the water 2.6 s later. How high was the platform, and how far from the edge of the platform did you hit the water? Ignore air resistance.

56. Archery An arrow is shot at 30.0° above the horizontal. Its velocity is 49 m/s, and it hits the target.
a. What is the maximum height the arrow will attain?
b. The target is at the height from which the arrow was shot. How far away is it?

57. Hitting a Home Run A pitched ball is hit by a batter at a 45° angle and just clears the outfield fence, 98 m away. If the fence is at the same height as the pitch, find the velocity of the ball when it left the bat. Ignore air resistance.

58. At-Sea Rescue An airplane traveling 1001 m above the ocean at 125 km/h is going to drop a box of supplies to shipwrecked victims below.
a. How many seconds before the plane is directly overhead should the box be dropped?
b. What is the horizontal distance between the plane and the victims when the box is dropped?

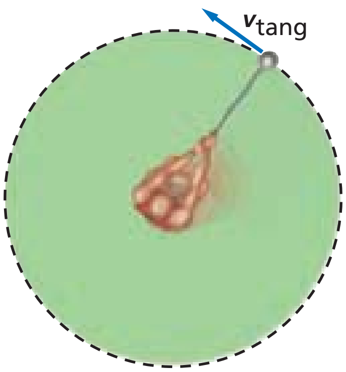
59. Diving Divers in Acapulco dive from a cliff that is 61 m high. If the rocks below the cliff extend outward for 23 m, what is the minimum horizontal velocity a diver must have to clear the rocks?

60. Jump Shot A basketball player is trying to make a half-court jump shot and releases the ball at the height of the basket. Assuming that the ball is launched at 51.0°, 14.0 m from the basket, what speed must the player give the ball?

6.2 Circular Motion

61. Car Racing A 615-kg racing car completes one lap in 14.3 s around a circular track with a radius of 50.0 m. The car moves at a constant speed.
  a. What is the acceleration of the car?
  b. What force must the track exert on the tires to produce this acceleration?

62. Hammer Throw An athlete whirls a 7.00-kg hammer 1.8 m from the axis of rotation in a horizontal circle, as shown in **Figure 6-14.** If the hammer makes one revolution in 1.0 s, what is the centripetal acceleration of the hammer? What is the tension in the chain?



■ Figure 6-14

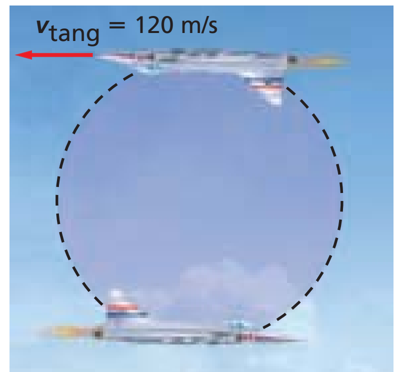
63. A coin is placed on a vinyl stereo record that is making $33\frac{1}{3}$ revolutions per minute on a turntable.
  a. In what direction is the acceleration of the coin?
  b. Find the magnitude of the acceleration when the coin is placed 5.0, 10.0, and 15.0 cm from the center of the record.
  c. What force accelerates the coin?
  d. At which of the three radii in part **b** would the coin be most likely to fly off the turntable? Why?

64. A rotating rod that is 15.3 cm long is spun with its axis through one end of the rod so that the other end of the rod has a speed of 2010 m/s (4500 mph).
  a. What is the centripetal acceleration of the end of the rod?
  b. If you were to attach a 1.0-g object to the end of the rod, what force would be needed to hold it on the rod?

65. Friction provides the force needed for a car to travel around a flat, circular race track. What is the maximum speed at which a car can safely travel if the radius of the track is 80.0 m and the coefficient of friction is 0.40?

66. A carnival clown rides a motorcycle down a ramp and around a vertical loop. If the loop has a radius of 18 m, what is the slowest speed the rider can have at the top of the loop to avoid falling? *Hint: At this slowest speed, the track exerts no force on the motorcycle at the top of the loop.*

67. A 75-kg pilot flies a plane in a loop as shown in **Figure 6-15.** At the top of the loop, when the plane is completely upside-down for an instant, the pilot hangs freely in the seat and does not push against the seat belt. The airspeed indicator reads 120 m/s. What is the radius of the plane's loop?



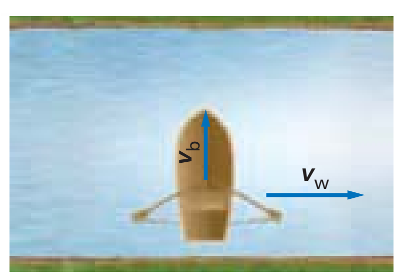
■ Figure 6-15

6.3 Relative Velocity

68. Navigating an Airplane An airplane flies at 200.0 km/h relative to the air. What is the velocity of the plane relative to the ground if it flies during the following wind conditions?
  a. a 50.0-km/h tailwind
  b. a 50.0-km/h headwind

69. Odina and LaToya are sitting by a river and decide to have a race. Odina will run down the shore to a dock, 1.5 km away, then turn around and run back. LaToya will also race to the dock and back, but she will row a boat in the river, which has a current of 2.0 m/s. If Odina's running speed is equal to LaToya's rowing speed in still water, which is 4.0 m/s, who will win the race? Assume that they both turn instantaneously.

70. Crossing a River You row a boat, such as the one in **Figure 6-16,** perpendicular to the shore of a river that flows at 3.0 m/s. The velocity of your boat is 4.0 m/s relative to the water.
  a. What is the velocity of your boat relative to the shore?
  b. What is the component of your velocity parallel to the shore? Perpendicular to it?



■ Figure 6-16

71. **Studying the Weather** A weather station releases a balloon to measure cloud conditions that rises at a constant 15 m/s relative to the air, but there is also a wind blowing at 6.5 m/s toward the west. What are the magnitude and direction of the velocity of the balloon?

72. **Boating** You are boating on a river that flows toward the east. Because of your knowledge of physics, you head your boat 53° west of north and have a velocity of 6.0 m/s due north relative to the shore.

a. What is the velocity of the current?

b. What is the speed of your boat relative to the water?

73. **Air Travel** You are piloting a small plane, and you want to reach an airport 450 km due south in 3.0 h. A wind is blowing from the west at 50.0 km/h. What heading and airspeed should you choose to reach your destination in time?

Mixed Review

74. Early skeptics of the idea of a rotating Earth said that the fast spin of Earth would throw people at the equator into space. The radius of Earth is about 6.38×10^3 km. Show why this idea is wrong by calculating the following.

a. the speed of a 97-kg person at the equator

b. the force needed to accelerate the person in the circle

c. the weight of the person

d. the normal force of Earth on the person, that is, the person's apparent weight

75. **Firing a Missile** An airplane, moving at 375 m/s relative to the ground, fires a missile forward at a speed of 782 m/s relative to the plane. What is the speed of the missile relative to the ground?

76. **Rocketry** A rocket in outer space that is moving at a speed of 1.25 km/s relative to an observer fires its motor. Hot gases are expelled out the back at 2.75 km/s relative to the rocket. What is the speed of the gases relative to the observer?

77. Two dogs, initially separated by 500.0 m, are running towards each other, each moving with a constant speed of 2.5 m/s. A dragonfly, moving with a constant speed of 3.0 m/s, flies from the nose of one dog to the other, then turns around instantaneously and flies back to the other dog. It continues to fly back and forth until the dogs run into each other. What distance does the dragonfly fly during this time?

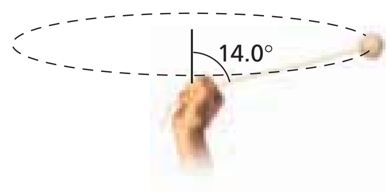
78. A 1.13-kg ball is swung vertically from a 0.50-m cord in uniform circular motion at a speed of 2.4 m/s. What is the tension in the cord at the bottom of the ball's motion?

79. **Banked Roads** Curves on roads often are banked to help prevent cars from slipping off the road. If the posted speed limit for a particular curve of radius 36.0 m is 15.7 m/s (35 mph), at what angle should the road be banked so that cars will stay on a circular path even if there were no friction between the road and the tires? If the speed limit was increased to 20.1 m/s (45 mph), at what angle should the road be banked?

80. The 1.45-kg ball in **Figure 6-17** is suspended from a 0.80-m string and swung in a horizontal circle at a constant speed such that the string makes an angle of 14.0° with the vertical.

a. What is the tension in the string?

b. What is the speed of the ball?



■ **Figure 6-17**

81. A baseball is hit directly in line with an outfielder at an angle of 35.0° above the horizontal with an initial velocity of 22.0 m/s. The outfielder starts running as soon as the ball is hit at a constant velocity of 2.5 m/s and barely catches the ball. Assuming that the ball is caught at the same height at which it was hit, what was the initial separation between the hitter and outfielder? *Hint: There are two possible answers.*

82. **A Jewel Heist** You are serving as a technical consultant for a locally produced cartoon. In one episode, two criminals, Shifty and Lefty, have stolen some jewels. Lefty has the jewels when the police start to chase him, and he runs to the top of a 60.0-m tall building in his attempt to escape. Meanwhile, Shifty runs to the convenient hot-air balloon 20.0 m from the base of the building and untethers it, so it begins to rise at a constant speed. Lefty tosses the bag of jewels horizontally with a speed of 7.3 m/s just as the balloon begins its ascent. What must the velocity of the balloon be for Shifty to easily catch the bag?

Thinking Critically

83. Apply Concepts Consider a roller-coaster loop like the one in **Figure 6-18.** Are the cars traveling through the loop in uniform circular motion? Explain.

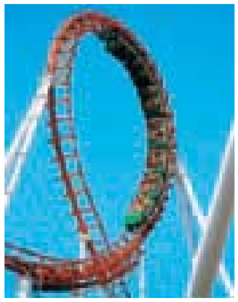


Figure 6-18

84. Use Numbers A 3-point jump shot is released 2.2 m above the ground and 6.02 m from the basket. The basket is 3.05 m above the floor. For launch angles of 30.0° and 60.0°, find the speed the ball needs to be thrown to make the basket.

85. Analyze For which angle in problem 84 is it more important that the player get the speed right? To explore this question, vary the speed at each angle by 5 percent and find the change in the range of the attempted shot.

86. Apply Computers and Calculators A baseball player hits a belt-high (1.0 m) fastball down the left-field line. The ball is hit with an initial velocity of 42.0 m/s at 26°. The left-field wall is 96.0 m from home plate at the foul pole and is 14-m high. Write the equation for the height of the ball, y, as a function of its distance from home plate, x. Use a computer or graphing calculator to plot the path of the ball. Trace along the path to find how high above the ground the ball is when it is at the wall.

a. Is the hit a home run?
b. What is the minimum speed at which the ball could be hit and clear the wall?
c. If the initial velocity of the ball is 42.0 m/s, for what range of angles will the ball go over the wall?

87. Analyze Albert Einstein showed that the rule you learned for the addition of velocities does not work for objects moving near the speed of light. For example, if a rocket moving at velocity v_A releases a missile that has velocity v_B relative to the rocket, then the velocity of the missile relative to an observer that is at rest is given by $v = (v_A + v_B)/(1 + v_A v_B/c^2)$, where c is the speed of light, 3.00×10^8 m/s. This formula gives the correct values for objects moving at slow speeds as well. Suppose a rocket moving at 11 km/s shoots a laser beam out in front of it. What speed would an unmoving observer find for the laser light? Suppose that a rocket moves at a speed $c/2$, half the speed of light, and shoots a missile forward at a speed of $c/2$ relative to the rocket. How fast would the missile be moving relative to a fixed observer?

88. Analyze and Conclude A ball on a light string moves in a vertical circle. Analyze and describe the motion of this system. Be sure to consider the effects of gravity and tension. Is this system in uniform circular motion? Explain your answer.

Writing in Physics

89. Roller Coasters If you take a look at vertical loops on roller coasters, you will notice that most of them are not circular in shape. Research why this is so and explain the physics behind this decision by the coaster engineers.

90. Many amusement-park rides utilize centripetal acceleration to create thrills for the park's customers. Choose two rides other than roller coasters that involve circular motion and explain how the physics of circular motion creates the sensations for the riders.

Cumulative Review

91. Multiply or divide, as indicated, using significant digits correctly. (Chapter 1)

a. $(5\times10^8\ \text{m})(4.2\times10^7\ \text{m})$
b. $(1.67\times10^{-2}\ \text{km})(8.5\times10^{-6}\ \text{km})$
c. $(2.6\times10^4\ \text{kg})/(9.4\times10^3\ \text{m}^3)$
d. $(6.3\times10^{-1}\ \text{m})/(3.8\times10^2\ \text{s})$

92. Plot the data in **Table 6-1** on a position-time graph. Find the average velocity in the time interval between 0.0 s and 5.0 s. (Chapter 3)

Table 6-1
Position v. Time

Clock Reading t (s)	Position d (m)
0.0	30
1.0	30
2.0	35
3.0	45
4.0	60
5.0	70

93. Carlos and his older brother Ricardo are at the grocery store. Carlos, with mass 17.0 kg, likes to hang on the front of the cart while Ricardo pushes it, even though both boys know this is not safe. Ricardo pushes the cart, with mass 12.4 kg, with his brother hanging on it such that they accelerate at a rate of 0.20 m/s^2. (Chapter 4)

a. With what force is Ricardo pushing?
b. What is the force the cart exerts on Carlos?

 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. A 1.60-m-tall girl throws a football at an angle of 41.0° from the horizontal and at an initial velocity of 9.40 m/s. How far away from the girl will it land?

- (A) 4.55 m
- (B) 5.90 m
- (C) 8.90 m
- (D) 10.5 m

2. A dragonfly is sitting on a merry-go-round 2.8 m from the center. If the tangential velocity of the ride is 0.89 m/s, what is the centripetal acceleration of the dragonfly?

- (A) 0.11 m/s^2
- (B) 0.28 m/s^2
- (C) 0.32 m/s^2
- (D) 2.2 m/s^2

3. The centripetal force on a 0.82-kg object on the end of a 2.0-m massless string being swung in a horizontal circle is 4.0 N. What is the tangential velocity of the object?

- (A) 2.8 m/s^2
- (B) 3.1 m/s^2
- (C) 4.9 m/s^2
- (D) 9.8 m/s^2

4. A 1000-kg car enters an 80.0-m-radius curve at 20.0 m/s. What centripetal force must be supplied by friction so the car does not skid?

- (A) 5.0 N
- (B) 2.5×10^2 N
- (C) 5.0×10^3 N
- (D) 1.0×10^3 N

5. A jogger on a riverside path sees a rowing team coming toward him. If the jogger is moving at 10 km/h, and the boat is moving at 20 km/h, how quickly does the jogger approach the boat?

- (A) 3 m/s
- (B) 8 m/s
- (C) 40 m/s
- (D) 100 m/s

6. What is the maximum height obtained by a 125-g apple that is slung from a slingshot at an angle of 78° from the horizontal with an initial velocity of 18 m/s?

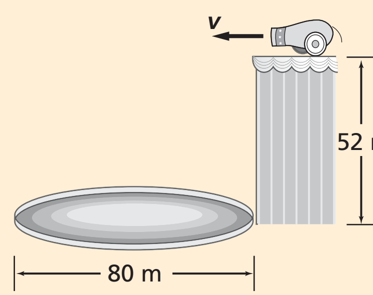
- (A) 0.70 m
- (B) 16 m
- (C) 32 m
- (D) 33 m

7. An orange is dropped at the same time a bullet is shot from a gun. Which of the following is true?

- (A) The acceleration due to gravity is greater for the orange because the orange is heavier.
- (B) Gravity acts less on the bullet than on the orange because the bullet is moving so fast.
- (C) The velocities will be the same.
- (D) The two objects will hit the ground at the same time.

Extended Answer

8. A colorfully feathered lead cannonball is shot horizontally out of a circus cannon 25 m/s from the high-wire platform on one side of a circus ring. If the high-wire platform is 52 m above the 80-m diameter ring, will the performers need to adjust their cannon (will the ball land inside the ring, or past it)?



(Not to scale.)

9. A mythical warrior swings a 5.6-kg mace on the end of a magically massless 86-cm chain in a horizontal circle above his head. The mace makes one full revolution in 1.8 s. Find the tension in the magical chain.

✓ Test-Taking TIP

Practice Under Testlike Conditions

Answer all of the questions in the time provided without referring to your book. Did you complete the test? Could you have made better use of your time? What topics do you need to review?