Chapter 9

Momentum and Its Conservation

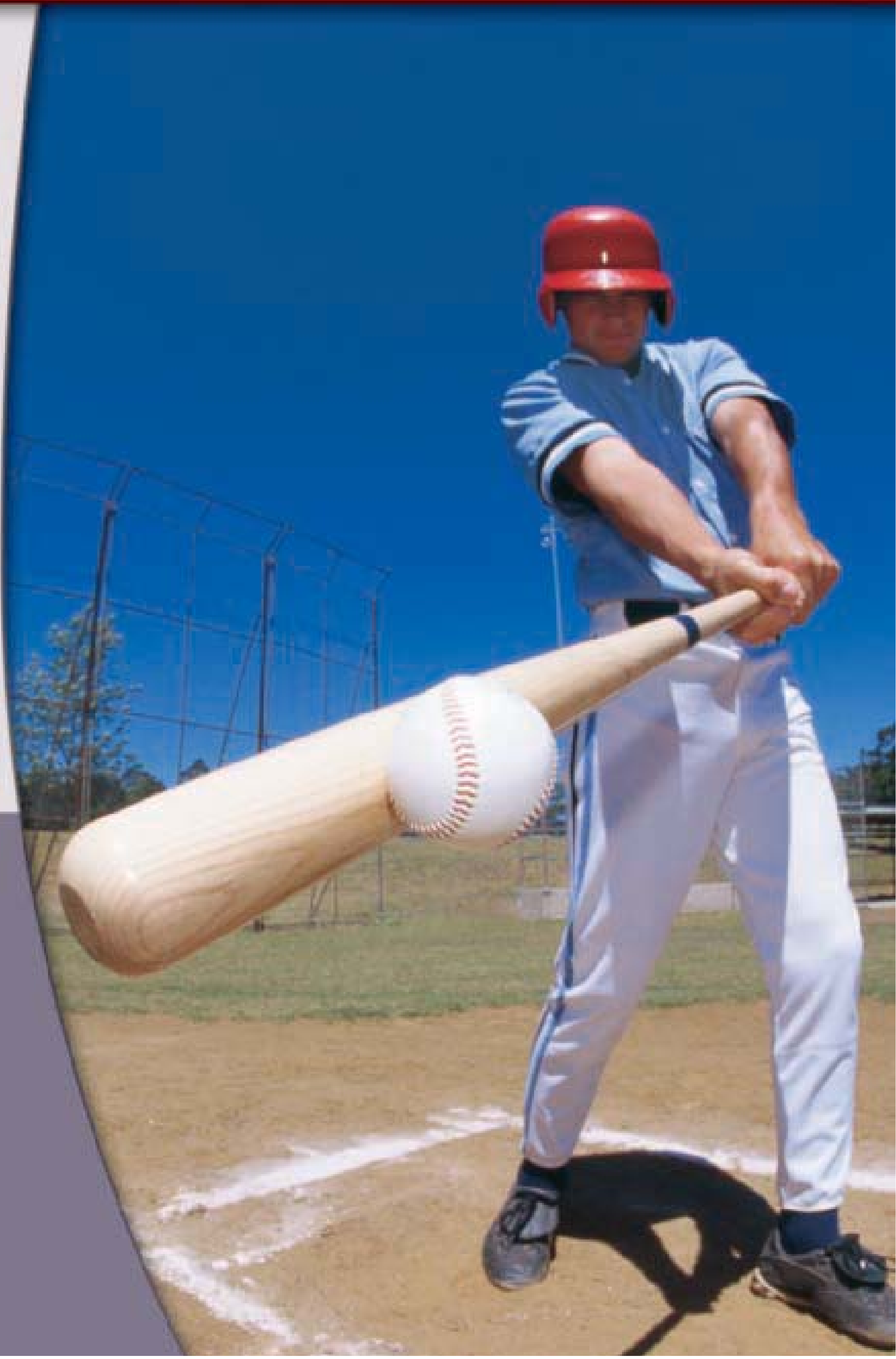
What You'll Learn

- You will describe momentum and impulse and apply them to the interactions between objects.
- You will relate Newton's third law of motion to conservation of momentum.
- You will explore the momentum of rotating objects.

Why It's Important

Momentum is the key to success in many sporting events, including baseball, football, ice hockey, and tennis.

Baseball Every baseball player dreams of hitting a home run. When a player hits the ball, at the moment of collision, the ball and the bat are deformed by the collision. The resulting change in momentum determines the batter's success.



Think About This ▶

What is the force on a baseball bat when a home run is hit out of the park?

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LAUNCH Lab

What happens when a hollow plastic ball strikes a bocce ball?

Question
What direction will a hollow plastic ball and a bocce ball move after a head-on collision?

Procedure

1. Roll a bocce ball and a hollow plastic ball toward each other on a smooth surface.
2. Observe the direction each one moves after the collision.
3. Repeat the experiment, this time keeping the bocce ball stationary, while rolling the hollow plastic ball toward it.
4. Observe the direction each one moves after the collision.
5. Repeat the experiment one more time, but keep the hollow plastic ball stationary, while rolling the bocce ball toward it.
6. Observe the direction each one moves after the collision.

Analysis
What factors affect how fast the balls move after the collision? What factors determine the direction each one moves after the collision?

Critical Thinking What factor(s) would cause the bocce ball to move backward after colliding with the hollow plastic ball?

9.1 Impulse and Momentum

It is always exciting to watch a baseball player hit a home run. The pitcher fires the baseball toward the plate. The batter swings at the baseball and the baseball recoils from the impact of the bat at high speed. Rather than concentrating on the force between the baseball and bat and their resulting accelerations, as in previous chapters, you will approach this collision in a different way in this chapter. The first step in analyzing this type of interaction is to describe what happens before, during, and after the collision between the baseball and bat. You can simplify the collision between the baseball and the bat by making the assumption that all motion is in the horizontal direction. Before the collision, the baseball moves toward the bat. During the collision, the baseball is squashed against the bat. After the collision, however, the baseball moves at a higher velocity away from the bat, and the bat continues in its path, but at a slower velocity.

Objectives
- **Define** the momentum of an object.
- **Determine** the impulse given to an object.
- **Define** the angular momentum of an object.

Vocabulary
impulse
momentum
impulse-momentum theorem
angular momentum
angular impulse-angular momentum theorem

Horizons Companies

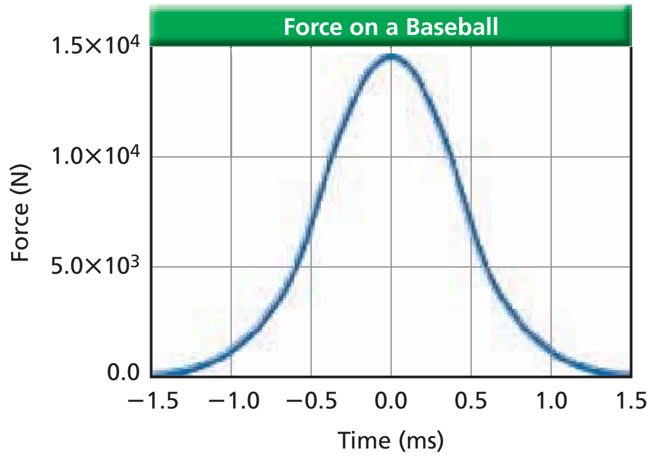

Figure 9-1 The force acting on a baseball increases, then rapidly decreases during a collision, as shown in this force-time graph.

Concepts In Motion

Interactive Figure To see an animation on impulse and momentum, visit physicspp.com.

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Impulse and Momentum

How are the velocities of the ball, before and after the collision, related to the force acting on it? Newton's second law of motion describes how the velocity of an object is changed by a net force acting on it. The change in velocity of the ball must have been caused by the force exerted by the bat on the ball. The force changes over time, as shown in **Figure 9-1.** Just after contact is made, the ball is squeezed, and the force increases. After the force reaches its maximum, which is more than 10,000 times the weight of the ball, the ball recovers its shape and snaps away from the bat. The force rapidly returns to zero. This whole event takes place within about 3.0 ms. How can you calculate the change in velocity of the baseball?

Impulse Newton's second law of motion, $\boldsymbol{F} = m\boldsymbol{a}$, can be rewritten by using the definition of acceleration as the change in velocity divided by the time needed to make that change. It can be represented by the following equation:

$$\boldsymbol{F} = m\boldsymbol{a} = m\left(\frac{\Delta \boldsymbol{v}}{\Delta t}\right)$$

Multiplying both sides of the equation by the time interval, Δt, results in the following equation:

$$\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v}$$

Impulse, or $\boldsymbol{F}\Delta t$, is the product of the average force on an object and the time interval over which it acts. Impulse is measured in newton-seconds. For instances in which the force varies with time, the magnitude of an impulse is found by determining the area under the curve of a force-time graph, such as the one shown in Figure 9-1.

The right side of the equation, $m\Delta\boldsymbol{v}$, involves the change in velocity: $\Delta\boldsymbol{v} = \boldsymbol{v}_f - \boldsymbol{v}_i$. Therefore, $m\Delta\boldsymbol{v} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$. The product of the object's mass, m, and the object's velocity, $\boldsymbol{v}$, is defined as the **momentum** of the object. Momentum is measured in kg·m/s. An object's momentum, also known as linear momentum, is represented by the following equation.

Momentum $\boldsymbol{p} = m\boldsymbol{v}$

The momentum of an object is equal to the mass of the object times the object's velocity.

Recall the equation $\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$. Because $m\boldsymbol{v}_f = \boldsymbol{p}_f$ and $m\boldsymbol{v}_i = \boldsymbol{p}_i$, this equation can be rewritten as follows: $\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v} = \boldsymbol{p}_f - \boldsymbol{p}_i$. The right side of this equation, $\boldsymbol{p}_f - \boldsymbol{p}_i$, describes the change in momentum of an object. Thus, the impulse on an object is equal to the change in its momentum, which is called the **impulse-momentum theorem.** The impulse-momentum theorem is represented by the following equation.

Impulse-Momentum Theorem $\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_i$

The impulse on an object is equal to the object's final momentum minus the object's initial momentum.

Color Convention

- Momentum and impulse vectors are **orange.**
- Force vectors are **blue.**
- Acceleration vectors are **violet.**
- Velocity vectors are **red.**
- Displacement vectors are **green.**

If the force on an object is constant, the impulse is the product of the force multiplied by the time interval over which it acts. Generally, the force is not constant, however, and the impulse is found by using an average force multiplied by the time interval over which it acts, or by finding the area under a force-time graph.

Because velocity is a vector, momentum also is a vector. Similarly, impulse is a vector because force is a vector. This means that signs will be important for motion in one dimension.

Using the Impulse-Momentum Theorem

What is the change in momentum of a baseball? From the impulse-momentum theorem, you know that the change in momentum is equal to the impulse acting on it. The impulse on a baseball can be calculated by using a force-time graph. In Figure 9-1, the area under the curve is approximately 13.1 N·s. The direction of the impulse is in the direction of the force. Therefore, the change in momentum of the ball also is 13.1 N·s. Because 1 N·s is equal to 1 kg·m/s, the momentum gained by the ball is 13.1 kg·m/s in the direction of the force acting on it.

Assume that a batter hits a fastball. Before the collision of the ball and bat, the ball, with a mass of 0.145 kg, has a velocity of −38 m/s. Assume that the positive direction is toward the pitcher. Therefore, the baseball's momentum is $p_i = (0.145\ \text{kg})(-38\ \text{m/s}) = -5.5\ \text{kg·m/s}$.

What is the momentum of the ball after the collision? Solve the impulse-momentum theorem for the final momentum: $p_f = p_i + F\Delta t$. The ball's final momentum is the sum of the initial momentum and the impulse. Thus, the ball's final momentum is calculated as follows.

$$p_f = p_i + 13.1\ \text{kg·m/s}$$
$$= -5.5\ \text{kg·m/s} + 13.1\ \text{kg·m/s} = +7.6\ \text{kg·m/s}$$

What is the baseball's final velocity? Because $p_f = mv_f$, solving for $\boldsymbol{v}_f$ yields the following:

$$v_f = \frac{p_f}{m} = \frac{+7.6\ \text{kg·m/s}}{+0.145\ \text{kg·m/s}} = +52\ \text{m/s}$$

A speed of 52 m/s is fast enough to clear most outfield fences if the baseball is hit in the correct direction.

Using the Impulse-Momentum Theorem to Save Lives

A large change in momentum occurs only when there is a large impulse. A large impulse can result either from a large force acting over a short period of time or from a smaller force acting over a long period of time.

What happens to the driver when a crash suddenly stops a car? An impulse is needed to bring the driver's momentum to zero. According to the impulse-momentum equation, $\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_i$. The final momentum, $\boldsymbol{p}_f$, is zero. The initial momentum, $\boldsymbol{p}_i$, is the same with or without an air bag. Thus, the impulse, $\boldsymbol{F}\Delta t$, also is the same. An air bag, such as the one shown in **Figure 9-2,** reduces the force by increasing the time interval during which it acts. It also exerts the force over a larger area of the person's body, thereby reducing the likelihood of injuries.

APPLYING PHYSICS

Running Shoes Running is hard on the feet. When a runner's foot strikes the ground, the force exerted by the ground on it is as much as four times the runner's weight. The cushioning in an athletic shoe is designed to reduce this force by lengthening the time interval over which the force is exerted.

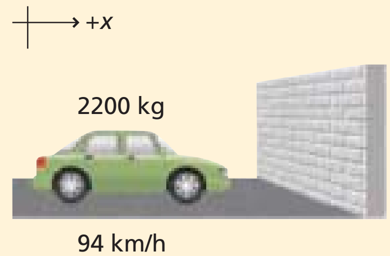
CLICK HERE

Figure 9-2 An air bag is inflated during a collision when the force due to the impact triggers the sensor. The chemicals in the air bag's inflation system react and produce a gas that rapidly inflates the air bag.

Rick Fischer/Masterfile

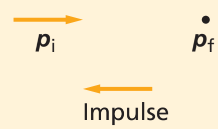
EXAMPLE Problem 1

Average Force A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21 s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes, or in 0.22 s if it hits a concrete wall. What average force is exerted on the vehicle in each of these stops?



1 Analyze and Sketch the Problem

- Sketch the system.
- Include a coordinate axis and select the positive direction to be the direction of the velocity of the car.
- Draw a vector diagram for momentum and impulse.



Known:

m = 2200 kg
$\boldsymbol{v}_i$ = +26 m/s
$\boldsymbol{v}_f$ = +0.0 m/s
$\Delta t_{\text{gentle braking}}$ = 21 s
$\Delta t_{\text{hard braking}}$ = 3.8 s
$\Delta t_{\text{hitting a wall}}$ = 0.22 s

Unknown:

$\boldsymbol{F}_{\text{gentle braking}}$ = ?
$\boldsymbol{F}_{\text{hard braking}}$ = ?
$\boldsymbol{F}_{\text{hitting a wall}}$ = ?

2 Solve for the Unknown

Determine the initial momentum, p_i.

$$p_i = mv_i$$
$$= (2200\text{ kg})(+26\text{ m/s})$$ **Substitute m = 2200 kg, v_i = +26 m/s**
$$= +5.7\times10^4\text{ kg·m/s}$$

Determine the final momentum, p_f.

$$p_f = mv_f$$
$$= (2200\text{ kg})(+0.0\text{ m/s})$$ **Substitute m = 2200 kg, v_f = +0.0 m/s**
$$= +0.0\text{ kg·m/s}$$

Apply the impulse-momentum theorem to obtain the force needed to stop the vehicle.

$$F\Delta t = p_f - p_i$$
$$F\Delta t = (+0.0\text{ kg·m/s}) - (5.7\times10^4\text{ kg·m/s})$$ **Substitute p_f = 0.0 kg·m/s, p_i = 5.7×10⁴ kg·m/s**
$$= -5.7\times10^4\text{ kg·m/s}$$
$$F = \frac{-5.7\times10^4\text{ kg·m/s}}{\Delta t}$$

$$F_{\text{gentle braking}} = \frac{-5.7\times10^4\text{ kg·m/s}}{21\text{ s}}$$ **Substitute $\Delta t_{\text{gentle braking}}$ = 21 s**
$$= -2.7\times10^3\text{ N}$$

$$F_{\text{hard braking}} = \frac{-5.7\times10^4\text{ kg·m/s}}{3.8\text{ s}}$$ **Substitute $\Delta t_{\text{hard braking}}$ = 3.8 s**
$$= -1.5\times10^4\text{ N}$$

$$F_{\text{hitting a wall}} = \frac{-5.7\times10^4\text{ kg·m/s}}{0.22\text{ s}}$$ **Substitute $\Delta t_{\text{hitting a wall}}$ = 0.22 s**
$$= -2.6\times10^5\text{ N}$$

Math Handbook
Operations with Significant Digits
pages 835–836

3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Does the direction make sense?** Force is exerted in the direction opposite to the velocity of the car and thus, is negative.
- **Is the magnitude realistic?** People weigh hundreds of newtons, so it is reasonable that the force needed to stop a car would be in the thousands of newtons. The impulse is the same for all three stops. Thus, as the stopping time is shortened by more than a factor of 10, the force is increased by more than a factor of 10.

▶ PRACTICE Problems

• Additional Problems, Appendix B
• Solutions to Selected Problems, Appendix C

1. A compact car, with mass 725 kg, is moving at 115 km/h toward the east. Sketch the moving car.

a. Find the magnitude and direction of its momentum. Draw an arrow on your sketch showing the momentum.

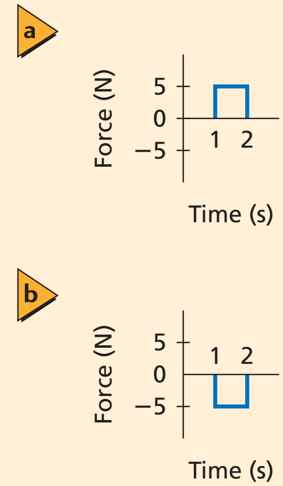
b. A second car, with a mass of 2175 kg, has the same momentum. What is its velocity?

2. The driver of the compact car in the previous problem suddenly applies the brakes hard for 2.0 s. As a result, an average force of 5.0×10^3 N is exerted on the car to slow it down.

a. What is the change in momentum; that is, the magnitude and direction of the impulse, on the car?

b. Complete the "before" and "after" sketches, and determine the momentum and the velocity of the car now.

3. A 7.0-kg bowling ball is rolling down the alley with a velocity of 2.0 m/s. For each impulse, shown in **Figures 9-3a** and **9-3b,** find the resulting speed and direction of motion of the bowling ball.



■ **Figure 9-3**

4. The driver accelerates a 240.0-kg snowmobile, which results in a force being exerted that speeds up the snowmobile from 6.00 m/s to 28.0 m/s over a time interval of 60.0 s.

a. Sketch the event, showing the initial and final situations.

b. What is the snowmobile's change in momentum? What is the impulse on the snowmobile?

c. What is the magnitude of the average force that is exerted on the snowmobile?

5. Suppose a 60.0-kg person was in the vehicle that hit the concrete wall in Example Problem 1. The velocity of the person equals that of the car both before and after the crash, and the velocity changes in 0.20 s. Sketch the problem.

a. What is the average force exerted on the person?

b. Some people think that they can stop their bodies from lurching forward in a vehicle that is suddenly braking by putting their hands on the dashboard. Find the mass of an object that has a weight equal to the force you just calculated. Could you lift such a mass? Are you strong enough to stop your body with your arms?

Angular Momentum

As you learned in Chapter 8, the angular velocity of a rotating object changes only if torque is applied to it. This is a statement of Newton's law for rotational motion, $\tau = I\Delta\omega/\Delta t$. This equation can be rearranged in the same way as Newton's second law of motion was, to produce $\tau\Delta t = I\Delta\omega$.

The left side of this equation, $\tau\Delta t$, is the angular impulse of the rotating object. The right side can be rewritten as $\Delta\omega = \omega_f - \omega_i$. The product of a rotating object's moment of inertia and angular velocity is called **angular momentum,** which is represented by the symbol L. The angular momentum of an object can be represented by the following equation.

Angular Momentum $L = I\omega$

The angular momentum of an object is equal to the product of the object's moment of inertia and the object's angular velocity.

Angular momentum is measured in kg·m^2/s. Just as the linear momentum of an object changes when an impulse acts on it, the angular momentum of an object changes when an angular impulse acts on it. Thus, the angular impulse on the object is equal to the change in the object's angular momentum, which is called the **angular impulse-angular momentum theorem.** The angular impulse-angular momentum theorem is represented by the following equation.

Angular Impulse-Angular Momentum Theorem $\tau\Delta t = L_f - L_i$

The angular impulse on an object is equal to the object's final angular momentum minus the object's initial angular momentum.

If there are no forces acting on an object, its linear momentum is constant. If there are no torques acting on an object, its angular momentum is also constant. Because an object's mass cannot be changed, if its momentum is constant, then its velocity is also constant. In the case of angular momentum, however, the object's angular velocity does not remain constant. This is because the moment of inertia depends on the object's mass and the way it is distributed about the axis of rotation or revolution. Thus, the angular velocity of an object can change even if no torques are acting on it.

Astronomy Connection

Consider, for example, a planet orbiting the Sun. The torque on the planet is zero because the gravitational force acts directly toward the Sun. Therefore, the planet's angular momentum is constant. When the distance between the planet and the Sun decreases, however, the planet's moment of inertia of revolution in orbit about the Sun also decreases. Thus, the planet's angular velocity increases and it moves faster. This is an explanation of Kepler's second law of planetary motion, based on Newton's laws of motion.

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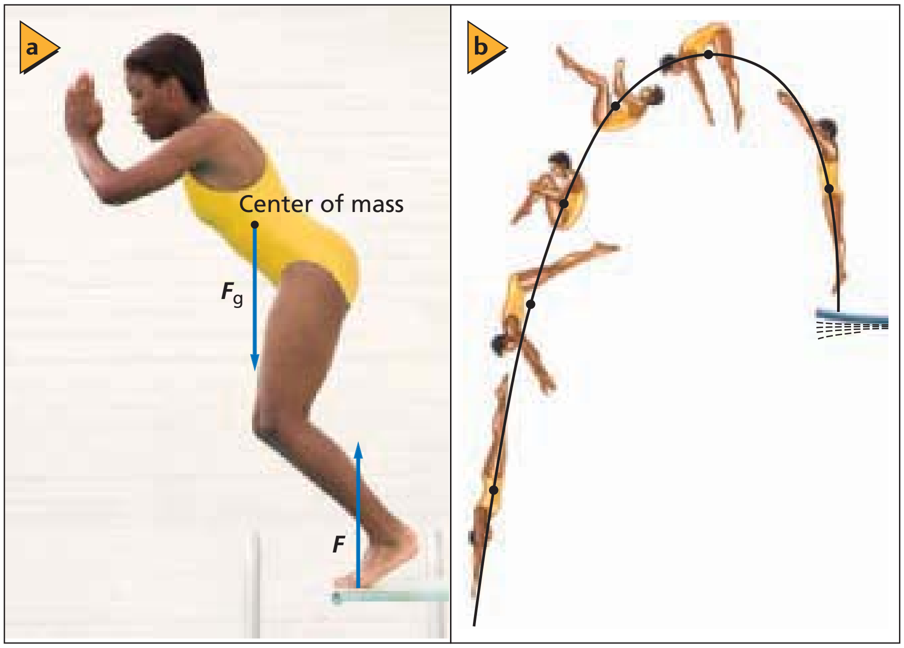
Figure 9-4 The diver's center of mass is in front of her feet as she gets ready to dive **(a).** As the diver changes her moment of inertia by moving her arms and legs to increase her angular momentum, the location of the center of mass changes, but the path of the center of mass remains a parabola **(b).**



Tim Fuller

Consider the diver in **Figure 9-4.** How does she start rotating her body? She uses the diving board to apply an external torque to her body. Then, she moves her center of mass in front of her feet and uses the board to give a final upward push to her feet. This torque acts over time, Δt, and thus increases the angular momentum of the diver.

Before the diver reaches the water, she can change her angular velocity by changing her moment of inertia. She may go into a tuck position, grabbing her knees with her hands. By moving her mass closer to the axis of rotation, the diver decreases her moment of inertia and increases her angular velocity. When she nears the water, she stretches her body straight, thereby increasing the moment of inertia and reducing the angular velocity. As a result, she goes straight into the water.

An ice-skater uses a similar method to spin. To begin rotating on one foot, the ice-skater applies an external torque to her body by pushing a portion of the other skate into the ice, as shown in **Figure 9-5.** If she pushes on the ice in one direction, the ice will exert a force on her in the opposite direction. The force results in a torque if the force is exerted some distance away from the pivot point, and in a direction that is not toward it. The greatest torque for a given force will result if the push is perpendicular to the lever arm.

The ice-skater then can control her angular velocity by changing her moment of inertia. Both arms and one leg can be extended from the body to slow the rotation, or pulled in close to the axis of rotation to speed it up. To stop spinning, another torque must be exerted by using the second skate to create a way for the ice to exert the needed force.

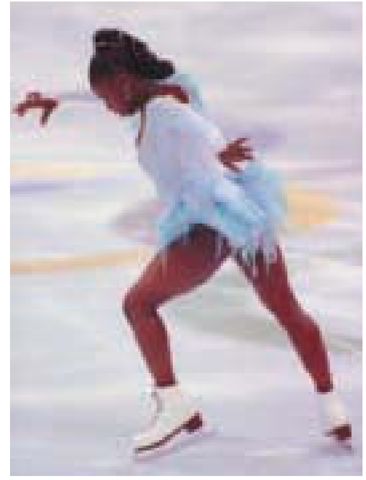


Figure 9-5 To spin on one foot, an ice-skater extends one leg and pushes on the ice. The ice exerts an equal and opposite force on her body and produces an external torque.

9.1 Section Review

6. Momentum Is the momentum of a car traveling south different from that of the same car when it travels north at the same speed? Draw the momentum vectors to support your answer.

7. Impulse and Momentum When you jump from a height to the ground, you let your legs bend at the knees as your feet hit the floor. Explain why you do this in terms of the physics concepts introduced in this chapter.

8. Momentum Which has more momentum, a supertanker tied to a dock or a falling raindrop?

9. Impulse and Momentum A 0.174-kg softball is pitched horizontally at 26.0 m/s. The ball moves in the opposite direction at 38.0 m/s after it is hit by the bat.

a. Draw arrows showing the ball's momentum before and after the bat hits it.

b. What is the change in momentum of the ball?

c. What is the impulse delivered by the bat?

d. If the bat and softball are in contact for 0.80 ms, what is the average force that the bat exerts on the ball?

10. Momentum The speed of a basketball as it is dribbled is the same when the ball is going toward the floor as it is when the ball rises from the floor. Is the basketball's change in momentum equal to zero when it hits the floor? If not, in which direction is the change in momentum? Draw the basketball's momentum vectors before and after it hits the floor.

11. Angular Momentum An ice-skater spins with his arms outstretched. When he pulls his arms in and raises them above his head, he spins much faster than before. Did a torque act on the ice-skater? If not, how could his angular velocity have increased?

12. Critical Thinking An archer shoots arrows at a target. Some of the arrows stick in the target, while others bounce off. Assuming that the masses of the arrows and the velocities of the arrows are the same, which arrows produce a bigger impulse on the target? *Hint: Draw a diagram to show the momentum of the arrows before and after hitting the target for the two instances.*

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9.2 Conservation of Momentum

Objectives

- **Relate** Newton's third law to conservation of momentum.
- **Recognize** the conditions under which momentum is conserved.
- **Solve** conservation of momentum problems.

Vocabulary

closed system
isolated system
law of conservation of momentum
law of conservation of angular momentum

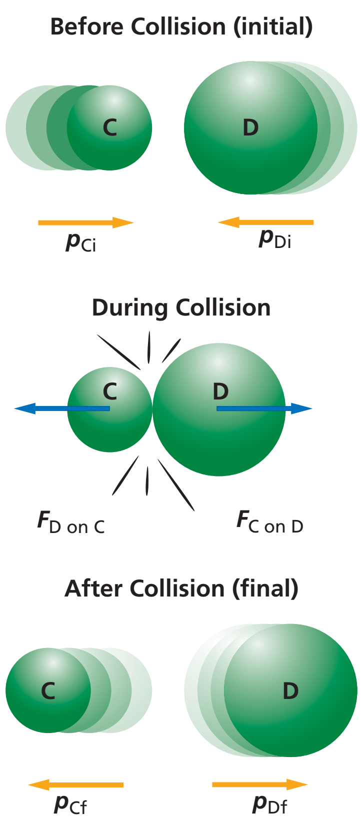


Figure 9-6 When two balls collide, they exert forces on each other that change their momenta.

In the first section of this chapter, you learned how a force applied during a time interval changes the momentum of a baseball. In the discussion of Newton's third law of motion, you learned that forces are the result of interactions between two objects. The force of a bat on a ball is accompanied by an equal and opposite force of the ball on the bat. Does the momentum of the bat, therefore, also change?

Two-Particle Collisions

The bat, the hand and arm of the batter, and the ground on which the batter is standing are all objects that interact when a batter hits the ball. Thus, the bat cannot be considered a single object. In contrast to this complex system, examine for a moment the much simpler system shown in **Figure 9-6,** the collision of two balls.

During the collision of the two balls, each one briefly exerts a force on the other. Despite the differences in sizes and velocities of the balls, the forces that they exert on each other are equal and opposite, according to Newton's third law of motion. These forces are represented by the following equation: $\boldsymbol{F}_{\text{D on C}} = -\boldsymbol{F}_{\text{C on D}}$

How do the impulses imparted by both balls compare? Because the time intervals over which the forces are exerted are the same, the impulses must be equal in magnitude but opposite in direction. How did the momenta of the balls change as a result of the collision?

According to the impulse-momentum theorem, the change in momentum is equal to the impulse. Compare the changes in the momenta of the two balls.

$$\text{For ball C: } \boldsymbol{p}_{\text{Cf}} - \boldsymbol{p}_{\text{Ci}} = \boldsymbol{F}_{\text{D on C}}\Delta t$$

$$\text{For ball D: } \boldsymbol{p}_{\text{Df}} - \boldsymbol{p}_{\text{Di}} = \boldsymbol{F}_{\text{C on D}}\Delta t$$

Because the time interval over which the forces were exerted is the same, the impulses are equal in magnitude, but opposite in direction. According to Newton's third law of motion, $-\boldsymbol{F}_{\text{C on D}} = \boldsymbol{F}_{\text{D on C}}$. Thus,

$$\boldsymbol{p}_{\text{Cf}} - \boldsymbol{p}_{\text{Ci}} = -(\boldsymbol{p}_{\text{Df}} - \boldsymbol{p}_{\text{Di}}), \text{ or } \boldsymbol{p}_{\text{Cf}} + \boldsymbol{p}_{\text{Df}} = \boldsymbol{p}_{\text{Ci}} + \boldsymbol{p}_{\text{Di}}.$$

This equation states that the sum of the momenta of the balls is the same before and after the collision. That is, the momentum gained by ball D is equal to the momentum lost by ball C. If the system is defined as the two balls, the momentum of the system is constant, and therefore, momentum is conserved for the system.

Momentum in a Closed, Isolated System

Under what conditions is the momentum of the system of two balls conserved? The first and most obvious condition is that no balls are lost and no balls are gained. Such a system, which does not gain or lose mass, is said to be a **closed system.** The second condition required to conserve the momentum of a system is that the forces involved are internal forces; that is, there are no forces acting on the system by objects outside of it.

When the net external force on a closed system is zero, the system is described as an **isolated system.** No system on Earth can be said to be absolutely isolated, however, because there will always be some interactions between a system and its surroundings. Often, these interactions are small enough to be ignored when solving physics problems.

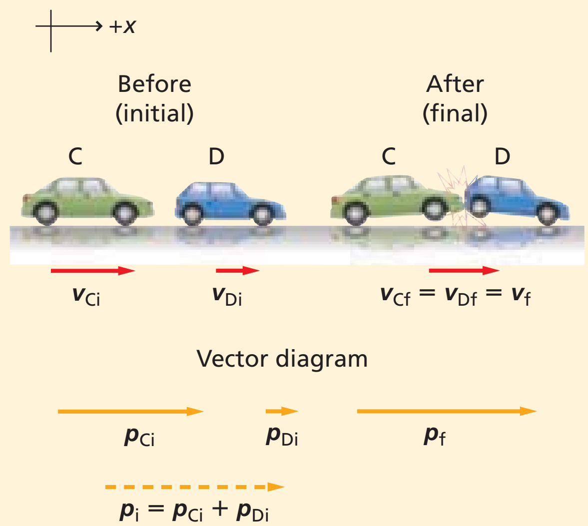
Systems can contain any number of objects, and the objects can stick together or come apart in a collision. Under these conditions, the **law of conservation of momentum** states that the momentum of any closed, isolated system does not change. This law will enable you to make a connection between conditions, before and after an interaction, without knowing any of the details of the interaction.

EXAMPLE Problem 2

Speed A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two cars move together immediately after the collision?

1 Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate system.
- Sketch the situation showing the "before" and "after" states.
- Draw a vector diagram for the momentum.



Known:

$m_C = 1875$ kg

$\mathbf{v}_{Ci} = +23$ m/s

$m_D = 1025$ kg

$\mathbf{v}_{Di} = +17$ m/s

Unknown:

$\mathbf{v}_f = ?$

2 Solve for the Unknown

Momentum is conserved because the ice makes the total external force on the cars nearly zero.

$$p_i = p_f$$

$$p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$$

$$m_C v_{Ci} + m_D v_{Di} = m_C v_{Cf} + m_D v_{Df}$$

Because the two cars stick together, their velocities after the collision, denoted as v_f, are equal.

$$v_{Cf} = v_{Df} = v_f$$

$$m_C v_{Ci} + m_D v_{Di} = (m_C + m_D) v_f$$

Solve for v_f.

$$v_f = \frac{(m_C v_{Ci} + m_D v_{Di})}{(m_C + m_D)}$$

$$= \frac{(1875\text{ kg})(+23\text{ m/s}) + (1025\text{ kg})(+17\text{ m/s})}{(1875\text{ kg} + 1025\text{ kg})}$$

Substitute $m_C = 1875$ kg, $v_{Ci} = +23$ m/s, $m_D = 1025$ kg, $v_{Di} = +17$ m/s

$$= +21\text{ m/s}$$

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3 Evaluate the Answer

- **Are the units correct?** Velocity is measured in m/s.
- **Does the direction make sense?** v_i and v_f are in the positive direction; therefore, v_f should be positive.
- **Is the magnitude realistic?** The magnitude of v_f is between the initial speeds of the two cars, but closer to the speed of the more massive one, so it is reasonable.

PRACTICE Problems

• Additional Problems, Appendix B
• Solutions to Selected Problems, Appendix C

13. Two freight cars, each with a mass of 3.0×10^5 kg, collide and stick together. One was initially moving at 2.2 m/s, and the other was at rest. What is their final speed?

14. A 0.105-kg hockey puck moving at 24 m/s is caught and held by a 75-kg goalie at rest. With what speed does the goalie slide on the ice?

15. A 35.0-g bullet strikes a 5.0-kg stationary piece of lumber and embeds itself in the wood. The piece of lumber and bullet fly off together at 8.6 m/s. What was the original speed of the bullet?

16. A 35.0-g bullet moving at 475 m/s strikes a 2.5-kg bag of flour that is on ice, at rest. The bullet passes through the bag, as shown in **Figure 9-7,** and exits it at 275 m/s. How fast is the bag moving when the bullet exits?

17. The bullet in the previous problem strikes a 2.5-kg steel ball that is at rest. The bullet bounces backward after its collision at a speed of 5.0 m/s. How fast is the ball moving when the bullet bounces backward?

18. A 0.50-kg ball that is traveling at 6.0 m/s collides head-on with a 1.00-kg ball moving in the opposite direction at a speed of 12.0 m/s. The 0.50-kg ball bounces backward at 14 m/s after the collision. Find the speed of the second ball after the collision.

Figure 9-7

Recoil

It is very important to define a system carefully. The momentum of a baseball changes when the external force of a bat is exerted on it. The baseball, therefore, is not an isolated system. On the other hand, the total momentum of two colliding balls within an isolated system does not change because all forces are between the objects within the system.

Can you find the final velocities of the two in-line skaters in **Figure 9-8?** Assume that they are skating on a smooth surface with no external forces. They both start at rest, one behind the other.

CLICK HERE

Figure 9-8 The internal forces exerted by Skater C, the boy, and Skater D, the girl, cannot change the total momentum of the system.

Skater C, the boy, gives skater D, the girl, a push. Now, both skaters are moving, making this situation similar to that of an explosion. Because the push was an internal force, you can use the law of conservation of momentum to find the skaters' relative velocities. The total momentum of the system was zero before the push. Therefore, it must be zero after the push.

Before		After
$\boldsymbol{p}_{Ci} + \boldsymbol{p}_{Di}$	=	$\boldsymbol{p}_{Cf} + \boldsymbol{p}_{Df}$
0	=	$\boldsymbol{p}_{Cf} + \boldsymbol{p}_{Df}$
$\boldsymbol{p}_{Cf}$	=	$-\boldsymbol{p}_{Df}$
$m_C\boldsymbol{v}_{Cf}$	=	$-m_D\boldsymbol{v}_{Df}$

The coordinate system was chosen so that the positive direction is to the left. The momenta of the skaters after the push are equal in magnitude but opposite in direction. The backward motion of skater C is an example of recoil. Are the skaters' velocities equal and opposite? The last equation shown above, for the velocity of skater C, can be rewritten as follows:

$$\boldsymbol{v}_{Cf} = \left(\frac{-m_D}{m_C}\right)\boldsymbol{v}_{Df}$$

The velocities depend on the skaters' relative masses. If skater C has a mass of 68.0 kg and skater D's mass is 45.4 kg, then the ratio of their velocities will be 68.0 : 45.4, or 1.50. The less massive skater moves at the greater velocity. Without more information about how hard skater C pushed skater D, however, you cannot find the velocity of each skater.

Propulsion in Space

How does a rocket in space change its velocity? The rocket carries both fuel and oxidizer. When the fuel and oxidizer combine in the rocket motor, the resulting hot gases leave the exhaust nozzle at high speed. If the rocket and chemicals are the system, then the system is a closed system. The forces that expel the gases are internal forces, so the system is also an isolated system. Thus, objects in space can accelerate by using the law of conservation of momentum and Newton's third law of motion.

A NASA space probe, called *Deep Space 1,* performed a flyby of an asteroid a few years ago. The most unusual of the 11 new technologies on board was an ion engine that exerts as much force as a sheet of paper resting on a person's hand. The ion engine shown in **Figure 9-9,** operates differently from a traditional rocket engine. In a traditional rocket engine, the products of the chemical reaction taking place in the combustion chamber are released at high speed from the rear. In the ion engine, however, xenon atoms are expelled at a speed of 30 km/s, producing a force of only 0.092 N. How can such a small force create a significant change in the momentum of the probe? Instead of operating for only a few minutes, as the traditional chemical rockets do, the ion engine can run continuously for days, weeks, or months. Therefore, the impulse delivered by the engine is large enough to increase the momentum of the 490-kg spacecraft until it reaches the speed needed to complete its mission.

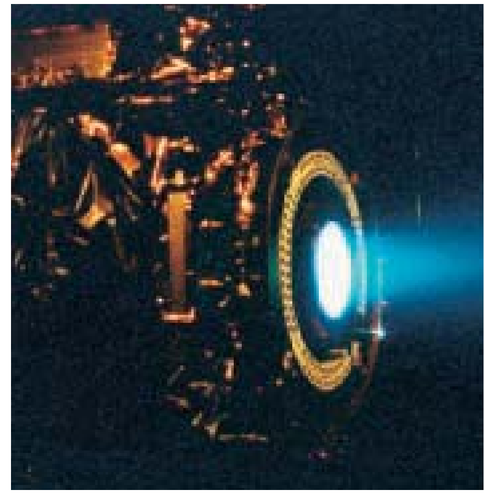


Figure 9-9 The xenon atoms in the ion engine are ionized by bombarding them with electrons. Then, the positively charged xenon ions are accelerated to high speeds.

MINI LAB

Rebound Height

An object's momentum is the product of its mass and velocity.

1. Drop a large rubber ball from about 15 cm above a table.

2. Measure and record the ball's rebound height.

3. Repeat steps 1–2 with a small rubber ball.

4. Hold the small rubber ball on top of, and in contact with, the large rubber ball.

5. Release the two rubber balls from the same height, so that they fall together.

6. Measure the rebound heights of both rubber balls.

Analyze and Conclude

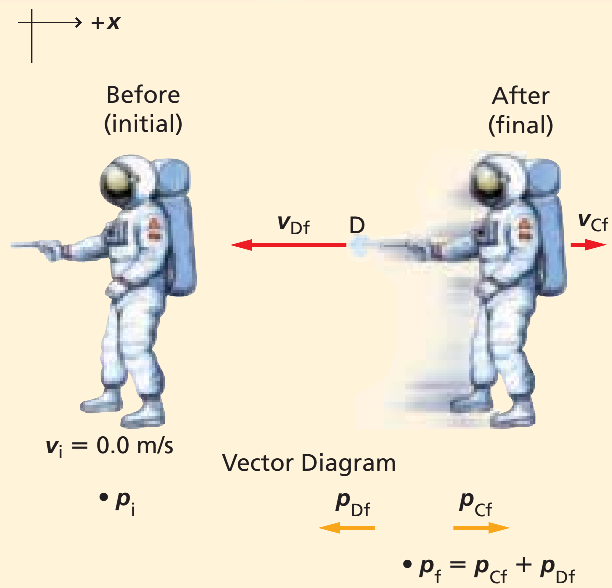
7. Describe the rebound height of each rubber ball dropped by itself.

8. Compare and contrast the rebound heights from number 7 with those from number 6.

9. Explain your observations.

EXAMPLE Problem 3

Speed An astronaut at rest in space fires a thruster pistol that expels 35 g of hot gas at 875 m/s. The combined mass of the astronaut and pistol is 84 kg. How fast and in what direction is the astronaut moving after firing the pistol?



1 Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate axis.
- Sketch the "before" and "after" conditions.
- Draw a vector diagram showing momenta.

Known:	Unknown:
m_C = 84 kg	v_{Cf} = ?
m_D = 0.035 kg	
$v_{Ci} = v_{Di}$ = +0.0 m/s	
v_{Df} = −875 m/s	

2 Solve for the Unknown

The system is the astronaut, the gun, and the chemicals that produce the gas.

$$p_i = p_{Ci} + p_{Di} = +0.0 \text{ kg·m/s}$$

Before the pistol is fired, all parts of the system are at rest; thus, the initial momentum is zero.

Use the law of conservation of momentum to find p_f.

$$p_i = p_f$$
$$+0.0 \text{ kg·m/s} = p_{Cf} + p_{Df}$$
$$p_{Cf} = -p_{Df}$$

The momentum of the astronaut is equal in magnitude, but opposite in direction to the momentum of the gas leaving the pistol.

Solve for the final velocity of the astronaut, v_{Cf}.

$$m_C v_{Cf} = -m_D v_{Df}$$
$$v_{Cf} = \left(\frac{-m_D v_{Df}}{m_C}\right)$$
$$= \frac{-(0.035 \text{ kg})(-875 \text{ m/s})}{84 \text{ kg}}$$

Substitute m_D = 0.035 kg, v_{Df} = −875 m/s, m_C = 84 kg

$$= +0.36 \text{ m/s}$$

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3 Evaluate the Answer

- **Are the units correct?** The velocity is measured in m/s.
- **Does the direction make sense?** The velocity of the astronaut is in the opposite direction to that of the expelled gas.
- **Is the magnitude realistic?** The astronaut's mass is much larger than that of the gas, so the velocity of the astronaut is much less than that of the expelled gas.

PRACTICE Problems

• Additional Problems, Appendix B
• Solutions to Selected Problems, Appendix C

19. A 4.00-kg model rocket is launched, expelling 50.0 g of burned fuel from its exhaust at a speed of 625 m/s. What is the velocity of the rocket after the fuel has burned? *Hint: Ignore the external forces of gravity and air resistance.*

20. A thread holds a 1.5-kg cart and a 4.5-kg cart together. After the thread is burned, a compressed spring pushes the carts apart, giving the 1.5-kg cart a speed of 27 cm/s to the left. What is the velocity of the 4.5-kg cart?

21. Carmen and Judi dock a canoe. 80.0-kg Carmen moves forward at 4.0 m/s as she leaves the canoe. At what speed and in what direction do the canoe and Judi move if their combined mass is 115 kg?

Two-Dimensional Collisions

Up until now, you have looked at momentum in only one dimension. The law of conservation of momentum holds for all closed systems with no external forces. It is valid regardless of the directions of the particles before or after they interact. But what happens in two or three dimensions? **Figure 9-10** shows the result of billiard ball C striking stationary billiard ball D. Consider the two billiard balls to be the system. The original momentum of the moving ball is $\boldsymbol{p}_{\mathrm{Ci}}$ and the momentum of the stationary ball is zero. Therefore, the momentum of the system before the collision is equal to $\boldsymbol{p}_{\mathrm{Ci}}$.

After the collision, both billiard balls are moving and have momenta. As long as the friction with the tabletop can be ignored, the system is closed and isolated. Thus, the law of conservation of momentum can be used. The initial momentum equals the vector sum of the final momenta, so $\boldsymbol{p}_{\mathrm{Ci}} = \boldsymbol{p}_{\mathrm{Cf}} + \boldsymbol{p}_{\mathrm{Df}}$.

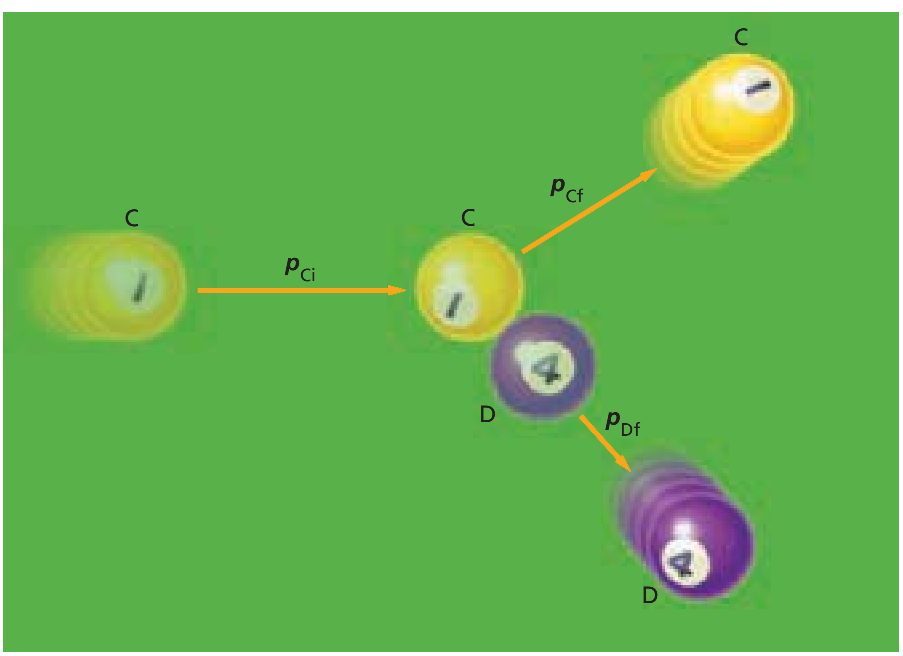
The equality of the momenta before and after the collision also means that the sum of the components of the vectors before and after the collision must be equal. Suppose the *x*-axis is defined to be in the direction of the initial momentum, then the *y*-component of the initial momentum is equal to zero. Therefore, the sum of the final *y*-components also must be zero:

$$\boldsymbol{p}_{\mathrm{Cf},\,y} + \boldsymbol{p}_{\mathrm{Df},\,y} = 0$$

The *y*-components are equal in magnitude but are in the opposite direction and, thus, have opposite signs. The sum of the horizontal components also is equal:

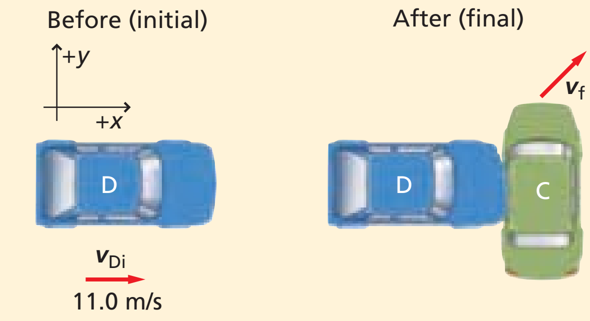
$$\boldsymbol{p}_{\mathrm{Ci}} = \boldsymbol{p}_{\mathrm{Cf},\,x} + \boldsymbol{p}_{\mathrm{Df},\,x}$$

Figure 9-10 The law of conservation of momentum holds for all isolated, closed systems, regardless of the directions of objects before and after a collision.



▶ EXAMPLE Problem 4

Speed A 1325-kg car, C, moving north at 27.0 m/s, collides with a 2165-kg car, D, moving east at 11.0 m/s. The two cars are stuck together. In what direction and with what speed do they move after the collision?

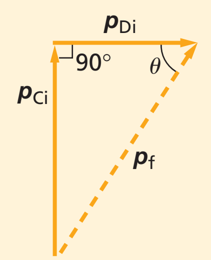


1 Analyze and Sketch the Problem

- Define the system.
- Sketch the "before" and "after" states.
- Establish the coordinate axis with the *y*-axis north and the *x*-axis east.
- Draw a momentum-vector diagram.

Known:	Unknown:
$m_C = 1325$ kg	$v_{f,x} = ?$
$m_D = 2165$ kg	$v_{f,y} = ?$
$v_{Ci,y} = 27.0$ m/s	$\theta = ?$
$v_{Di,x} = 11.0$ m/s	

Vector Diagram



2 Solve for the Unknown

Determine the initial momenta of the cars and the momentum of the system.

$$p_{Ci} = m_C v_{Ci,y}$$
$$= (1325\text{ kg})(27.0\text{ m/s}) \quad \textbf{Substitute } m_C = 1325\text{ kg}, v_{Ci,y} = 27.0\text{ m/s}$$
$$= 3.58\times10^4\text{ kg·m/s (north)}$$

$$p_{Di} = m_D v_{Di,x}$$
$$= (2165\text{ kg})(11.0\text{ m/s}) \quad \textbf{Substitute } m_D = 2165\text{ kg}, v_{Di,x} = 11.0\text{ m/s}$$
$$= 2.38\times10^4\text{ kg·m/s (east)}$$

Use the law of conservation of momentum to find p_f.

$$p_{f,x} = p_{i,x} = 2.38\times10^4\text{ kg·m/s} \quad \textbf{Substitute } p_{i,x} = p_{Di} = 2.38\times10^4\text{ kg·m/s}$$
$$p_{f,y} = p_{i,y} = 3.58\times10^4\text{ kg·m/s} \quad \textbf{Substitute } p_{i,y} = p_{Ci} = 3.58\times10^4\text{ kg·m/s}$$

Use the diagram to set up equations for $p_{f,x}$ and $p_{f,y}$.

$$p_f = \sqrt{(p_{f,x})^2 + (p_{f,y})^2}$$
$$= \sqrt{(2.38\times10^4\text{ kg·m/s})^2 + (3.58\times10^4\text{ kg·m/s})^2} \quad \textbf{Substitute } p_{f,x} = 2.38\times10^4\text{ kg·m/s}, p_{f,y} = 3.58\times10^4\text{ kg·m/s}$$
$$= 4.30\times10^4\text{ kg·m/s}$$

Solve for θ.

$$\theta = \tan^{-1}\left(\frac{p_{f,y}}{p_{f,x}}\right)$$
$$= \tan^{-1}\left(\frac{3.58\times10^4\text{ kg·m/s}}{2.38\times10^4\text{ kg·m/s}}\right) \quad \textbf{Substitute } p_{f,y} = 3.58\times10^4\text{ kg·m/s}, p_{f,x} = 2.38\times10^4\text{ kg·m/s}$$
$$= 56.4°$$

Determine the final speed.

$$v_f = \frac{p_f}{(m_C + m_D)}$$
$$= \frac{4.30\times10^4\text{ kg·m/s}}{(1325\text{ kg} + 2165\text{ kg})} \quad \textbf{Substitute } p_f = 4.30\times10^4\text{ kg·m/s}, m_C = 1325\text{ kg}, m_D = 2165\text{ kg}$$
$$= 12.3\text{ m/s}$$

3 Evaluate the Answer

- **Are the units correct?** The correct unit for speed is m/s.
- **Do the signs make sense?** Answers are both positive and at the appropriate angles.
- **Is the magnitude realistic?** The cars stick together, so v_f must be smaller than v_{Ci}.

Physics Online

Personal Tutor For an online tutorial on speed, visit physicspp.com.

PRACTICE Problems

- Additional Problems, Appendix B
- Solutions to Selected Problems, Appendix C

22. A 925-kg car moving north at 20.1 m/s collides with a 1865-kg car moving west at 13.4 m/s. The two cars are stuck together. In what direction and at what speed do they move after the collision?

23. A 1383-kg car moving south at 11.2 m/s is struck by a 1732-kg car moving east at 31.3 m/s. The cars are stuck together. How fast and in what direction do they move immediately after the collision?

24. A stationary billiard ball, with a mass of 0.17 kg, is struck by an identical ball moving at 4.0 m/s. After the collision, the second ball moves 60.0° to the left of its original direction. The stationary ball moves 30.0° to the right of the moving ball's original direction. What is the velocity of each ball after the collision?

25. A 1345-kg car moving east at 15.7 m/s is struck by a 1923-kg car moving north. They are stuck together and move with an initial velocity of 14.5 m/s at $\theta = 63.5°$. Was the north-moving car exceeding the 20.1 m/s speed limit?

Conservation of Angular Momentum

Like linear momentum, angular momentum can be conserved. The **law of conservation of angular momentum** states that if no net external torque acts on an object, then its angular momentum does not change. This is represented by the following equation.

Law of Conservation of Angular Momentum $L_i = L_f$

An object's initial angular momentum is equal to its final angular momentum.

For example, Earth spins on its axis with no external torques. Its angular momentum is constant. Thus, Earth's angular momentum is conserved. As a result, the length of a day does not change. A spinning ice-skater also demonstrates conservation of angular momentum. **Figure 9-11a** shows an ice-skater spinning with his arms extended. When he pulls in his arms, as shown in **Figure 9-11b,** he begins spinning faster. Without an external torque, his angular momentum does not change; that is, $L = I\omega$ is constant. Thus, the ice-skater's increased angular velocity must be accompanied by a decreased moment of inertia. By pulling his arms close to his body, the ice-skater brings more mass closer to the axis of rotation, thereby decreasing the radius of rotation and decreasing his moment of inertia. You can calculate changes in angular velocity using the law of conservation of angular momentum.

$$L_i = L_f$$

$$\text{thus, } I_i\omega_i = I_f\omega_f$$

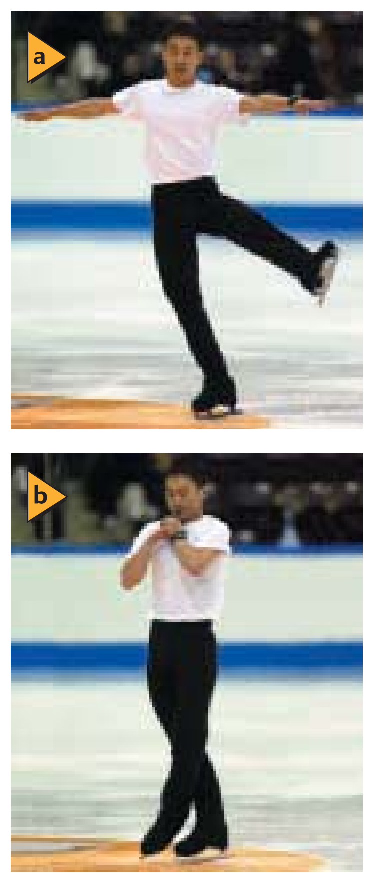
$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$

Because frequency is $f = \omega/2\pi$, the above equation can be rewritten as follows:

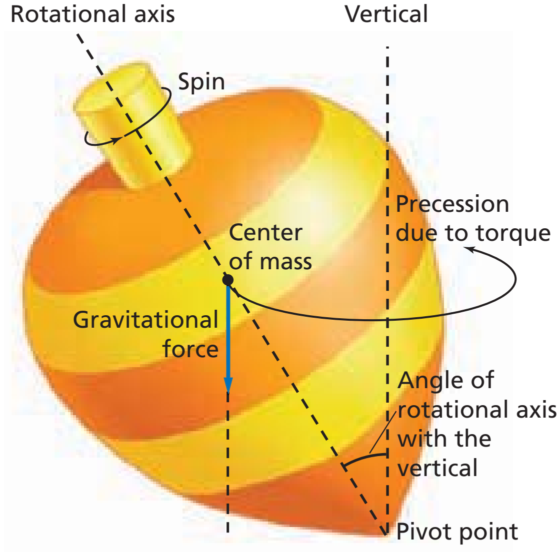
$$\frac{2\pi(f_f)}{2\pi(f_i)} = \frac{I_i}{I_f}$$

$$\text{thus, } \frac{f_f}{f_i} = \frac{I_i}{I_f}$$

Figure 9-11 When the ice-skater's arms are extended, the moment of inertia increases and his angular velocity decreases **(a).** When his arms are closer to his body the moment of inertia decreases and results in an increased angular velocity **(b).**



F. Scott Grant/IMAGE Communications



■ **Figure 9-12** The upper end of the top precesses due to the torque acting on the top.

Notice that because *f*, *ω*, and *I* appear as ratios in these equations, any units may be used, as long as the same unit is used for both values of the quantity.

If a torque-free object starts with no angular momentum, it must continue to have no angular momentum. Thus, if part of an object rotates in one direction, another part must rotate in the opposite direction. For example, if you switch on a loosely held electric drill, the drill body will rotate in the direction opposite to the rotation of the motor and bit.

Consider a ball thrown at a weather vane. The ball, moving in a straight line, can start the vane rotating. Consider the ball and vane to be a system. With no external torques, angular momentum is conserved. The vane spins faster if the ball has a large mass, *m*, a large velocity, *v*, and hits at right angles as far as possible from the pivot of the vane. The angular momentum of a moving object, such as the ball, is given by $L = mvr$, where *r* is the perpendicular distance from the axis of rotation.

Tops and Gyroscopes

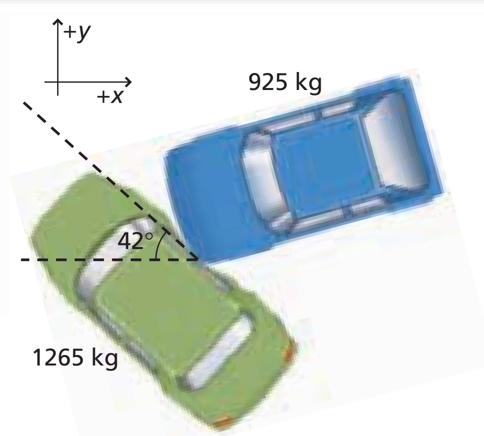
Because of the conservation of angular momentum, the direction of rotation of a spinning object can be changed only by applying a torque. If you played with a top as a child, you may have spun it by pulling the string wrapped around its axle. When a top is vertical, there is no torque on it, and the direction of its rotation does not change. If the top is tipped, as shown in **Figure 9-12,** a torque tries to rotate it downward. Rather than tipping over, however, the upper end of the top revolves, or precesses slowly about the vertical axis. Because Earth is not a perfect sphere, the Sun exerts a torque on it, causing it to precess. It takes about 26,000 years for Earth's rotational axis to go through one cycle of precession.

CHALLENGE PROBLEM

Your friend was driving her 1265-kg car north on Oak Street when she was hit by a 925-kg compact car going west on Maple Street. The cars stuck together and slid 23.1 m at 42° north of west. The speed limit on both streets is 22 m/s (50 mph). Assume that momentum was conserved during the collision and that acceleration was constant during the skid. The coefficient of kinetic friction between the tires and the pavement is 0.65.

1. Your friend claims that she wasn't speeding, but that the driver of other car was. How fast was your friend driving before the crash?

2. How fast was the other car moving before the crash? Can you support your friend's case in court?



A gyroscope, such as the one shown in **Figure 9-13,** is a wheel or disk that spins rapidly around one axis while being free to rotate around one or two other axes. The direction of its large angular momentum can be changed only by applying an appropriate torque. Without such a torque, the direction of the axis of rotation does not change. Gyroscopes are used in airplanes, submarines, and spacecraft to keep an unchanging reference direction. Giant gyroscopes are used in cruise ships to reduce their motion in rough water. Gyroscopic compasses, unlike magnetic compasses, maintain direction even when they are not on a level surface.

A football quarterback uses the gyroscope effect to make an accurate forward pass. As he throws, he spins, or spirals the ball. If the quarterback throws the ball in the direction of its spin axis of rotation, the ball keeps its pointed end forward, thereby reducing air resistance. Thus, the ball can be thrown far and accurately. If its spin direction is slightly off, the ball wobbles. If the ball is not spun, it tumbles end over end.

The flight of a plastic disk also is stabilized by spin. A well-spun plastic disk can fly many meters through the air without wobbling. You are able to perform tricks with a yo-yo because its fast rotational speed keeps it rotating in one plane.

■ **Figure 9-13** Because the orientation of the spin axis of the gyroscope does not change even when it is moved, the gyroscope can be used to fix direction.

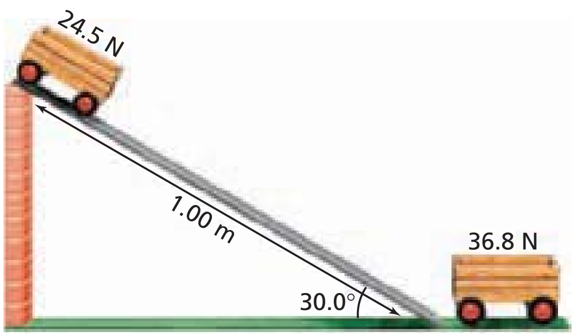
9.2 Section Review

26. Angular Momentum The outer rim of a plastic disk is thick and heavy. Besides making it easier to catch, how does this affect the rotational properties of the plastic disk?

27. Speed A cart, weighing 24.5 N, is released from rest on a 1.00-m ramp, inclined at an angle of 30.0° as shown in **Figure 9-14.** The cart rolls down the incline and strikes a second cart weighing 36.8 N.

a. Calculate the speed of the first cart at the bottom of the incline.

b. If the two carts stick together, with what initial speed will they move along?



■ **Figure 9-14**

28. Conservation of Momentum During a tennis serve, the racket of a tennis player continues forward after it hits the ball. Is momentum conserved in the collision? Explain, making sure that you define the system.

29. Momentum A pole-vaulter runs toward the launch point with horizontal momentum. Where does the vertical momentum come from as the athlete vaults over the crossbar?

30. Initial Momentum During a soccer game, two players come from opposite directions and collide when trying to head the ball. They come to rest in midair and fall to the ground. Describe their initial momenta.

31. Critical Thinking You catch a heavy ball while you are standing on a skateboard, and then you roll backward. If you were standing on the ground, however, you would be able to avoid moving while catching the ball. Explain both situations using the law of conservation of momentum. Explain which system you use in each case.

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Sticky Collisions

Alternate CBL instructions can be found on the Web site.
physicspp.com

In this activity, one moving cart will strike a stationary cart. During the collision, the two carts will stick together. You will measure mass and velocity, both before and after the collision. You then will calculate the momentum both before and after the collision.

QUESTION

How is the momentum of a system affected by a sticky collision?

Objectives

- **Describe** how momentum is transferred during a collision.
- **Calculate** the momenta involved.
- **Interpret data** from a collision.
- **Draw conclusions** that support the law of conservation of momentum.

Safety Precautions

Materials

Internet access required

Procedure

1. View Chapter 9 lab video clip 1 at physicspp.com/internet_lab to determine the mass of the carts.
2. Record the mass of each cart.
3. Watch video clip 2: Cart 1 strikes Cart 2.
4. In the video, three frames represent 0.1 s, and the main gridlines are separated by 10 cm. Record in the data table the distance Cart 1 travels in 0.1 s before the collision.
5. Observe the collision. Record in the data table the distance the Cart 1-Cart 2 system travels in 0.1 s after the collision.
6. Repeat steps 3–5 for video clip 3: Carts 1 and 3 strike Cart 2.
7. Repeat steps 3–5 for video clip 4: Cart 1 strikes Carts 2 and 3.
8. Repeat steps 3-5 for video clip 5: Carts 1 and 3 strike Carts 2 and 4.

Data Tables

Cart	Mass (kg)
1	
2	
3	
4	

Time of Approach (s)	Distance Covered in Approach (cm)	Initial Velocity (cm/s)	Mass of Approaching Cart(s) (g)	Initial Momentum (g·cm/s)	Time of Departure (s)	Distance Covered in Departure (cm)	Final Velocity (cm/s)	Mass of Departing Cart(s) (g)	Final Momentum (g·cm/s)
0.1					0.1				
0.1					0.1				
0.1					0.1				
0.1					0.1				

Analyze

1. Calculate the initial and final velocities for each of the cart systems.
2. Calculate the initial and final momentum for each of the cart systems.
3. **Make and Use Graphs** Make a graph showing final momentum versus initial momentum for all the video clips.

Conclude and Apply

1. What is the relationship between the initial momentum and the final momentum of the cart systems in a sticky collision?
2. In theory, what should be the slope of the line in your graph?
3. The initial and final data numbers may not be the same due to the precision of the instruments, friction, and other variables. Is the initial momentum typically greater or less than the final momentum? Explain.

Going Further

1. Describe what the velocity and momentum data might look like if the carts did not stick together, but rather, bounced off of each other.
2. Design an experiment to test the impact of friction during the collision of the cart systems. Predict how the slope of the line in your graph will change with your experiment, and then try your experiment.

Real-World Physics

1. Suppose a linebacker collides with a stationary quarterback and they become entangled. What will happen to the velocity of the linebacker-quarterback system if momentum is conserved?
2. If a car rear-ends a stationary car so that the two cars become attached, what will happen to the velocity of the first car? The second car?

Share Your Data

Interpret Data Visit physicspp.com/internet_lab to post your findings from the experiment testing the impact of friction during the collisions of the cart systems. Examine your data, and graph final momentum versus initial momentum. Notice how close to or far off the slope is from 1.00.

Physics Online

To find out more about momentum, visit the Web site: physicspp.com

Future Technology

Solar Sailing

Nearly 400 years ago, Johannes Kepler observed that comet tails appeared to be blown by a solar breeze. He suggested that ships would be able to travel in space with sails designed to catch this breeze. Thus, the idea for solar sails was born.

How Does a Solar Sail Work? A solar sail is a spacecraft without an engine. A solar sail works like a giant fabric mirror that is free to move. Solar sails usually are made of 5-micron-thick aluminized polyester film or polyimide film with a 100-nm-thick aluminum layer deposited on one side to form the reflective surface.

Reflected sunlight, rather than rocket fuel, provides the force. Sunlight is made up of individual particles called photons. Photons have momentum, and when a photon bounces off a solar sail, it transfers its momentum to the sail, which propels the spacecraft along.

This artist's rendering shows *Cosmos 1*, the first solar sail launched into space.

The force of impacting photons is small in comparison to the force rocket fuel can supply. So, small sails experience only a small amount of force from sunlight, while larger sails experience a greater force. Thus, solar sails may be a kilometer or so across.

What speeds can a solar sail achieve? This depends on the momentum transferred to the sail by photons, as well as the sail's mass. To travel quickly through the solar system, a sail and the spacecraft should be lightweight.

Photons supplied by the Sun are constant. They impact the sail every second of every hour of every day during a space flight. The Sun's continuous supply of photons over time allows the sail to build up huge velocities and enables the spacecraft to travel great distances within a convenient time frame. Rockets require enormous amounts of fuel to move large masses, but solar sails only require photons from the Sun. Thus, solar sails may be a superior way to move large masses over great distances in outer space.

Future Journeys The *Cosmos 1* mission, a privately-funded international venture, launched the first solar-sail prototype. *Cosmos 1* was launched toward space on the tip of a submarine-launched rocket on June 21, 2005. The spacecraft looked like a flower with eight huge, solar-sail petals. Even though the goals were modest, *Cosmos 1* never had the chance to test its new technology. The first stage of the rocket never completed its scheduled burn, preventing *Cosmos 1* from entering orbit.

Solar sails are important, not only for travel, but also for creating new types of space and Earth weather monitoring stations. These stations would be able to provide greater coverage of Earth and more advanced warning of solar storms that cause problems to communication and electric power grids. It is hoped that in the next few decades, solar sails will be used as interplanetary shuttles because of their ability to travel great distances in convenient time frames. Vast distances could someday be traversed by vehicles that do not consume any fuel.

Going Further

1. **Research** how solar sails can help provide advanced warning of solar storms.
2. **Critical Thinking** A certain solar-sail model is predicted to take more time to reach Mars than a rocket-propelled spacecraft would, but less time to go to Pluto than a rocket-propelled spacecraft would. Explain why this is so.

Chapter 9

Study Guide

9.1 Impulse and Momentum

Vocabulary

- impulse (p. 230)
- momentum (p. 230)
- impulse-momentum theorem (p. 230)
- angular momentum (p. 233)
- angular impulse-angular momentum theorem (p. 234)

Key Concepts

- When doing a momentum problem, first examine the system before and after the event.
- The momentum of an object is the product of its mass and velocity and is a vector quantity.

$$\boldsymbol{p} = m\boldsymbol{v}$$

- The impulse on an object is the average net force exerted on the object multiplied by the time interval over which the force acts.

$$\text{Impulse} = \boldsymbol{F}\Delta t$$

- The impulse on an object is equal to the change in momentum of the object.

$$\boldsymbol{F}\Delta t = \boldsymbol{p}_{\text{f}} - \boldsymbol{p}_{\text{i}}$$

- The angular momentum of a rotating object is the product of its moment of inertia and its angular velocity.

$$L = I\omega$$

- The angular impulse-angular momentum theorem states that the angular impulse on an object is equal to the change in the object's angular momentum.

$$\tau\Delta t = L_{\text{f}} - L_{\text{i}}$$

9.2 Conservation of Momentum

Vocabulary

- closed system (p. 236)
- isolated system (p. 237)
- law of conservation of momentum (p. 237)
- law of conservation of angular momentum (p. 243)

Key Concepts

- According to Newton's third law of motion and the law of conservation of momentum, the forces exerted by colliding objects on each other are equal in magnitude and opposite in direction.
- Momentum is conserved in a closed, isolated system.

$$\boldsymbol{p}_{\text{f}} = \boldsymbol{p}_{\text{i}}$$

- The law of conservation of momentum can be used to explain the propulsion of rockets.
- Vector analysis is used to solve momentum-conservation problems in two dimensions.
- The law of conservation of angular momentum states that if there are no external torques acting on a system, then the angular momentum is conserved.

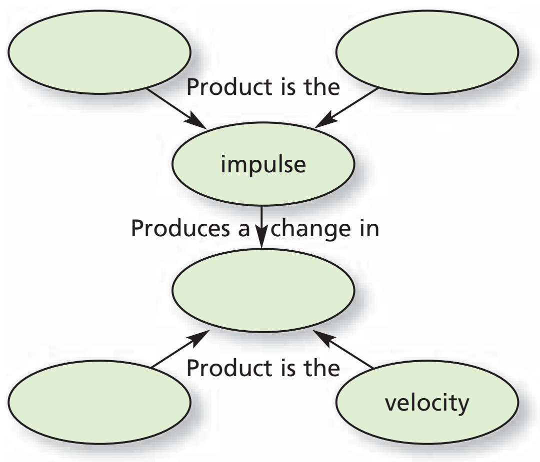
$$L_{\text{i}} = L_{\text{f}}$$

- Because angular momentum is conserved, the direction of rotation of a spinning object can be changed only by applying a torque.

Chapter 9 Assessment

Concept Mapping

32. Complete the following concept map using the following terms: *mass, momentum, average force, time over which the force is exerted.*



Mastering Concepts

33. Can a bullet have the same momentum as a truck? Explain. (9.1)

34. A pitcher throws a curve ball to the catcher. Assume that the speed of the ball doesn't change in flight. (9.1)
- **a.** Which player exerts the larger impulse on the ball?
- **b.** Which player exerts the larger force on the ball?

35. Newton's second law of motion states that if no net force is exerted on a system, no acceleration is possible. Does it follow that no change in momentum can occur? (9.1)

36. Why are cars made with bumpers that can be pushed in during a crash? (9.1)

37. An ice-skater is doing a spin. (9.1)
- **a.** How can the skater's angular momentum be changed?
- **b.** How can the skater's angular velocity be changed without changing the angular momentum?

38. What is meant by "an isolated system?" (9.2)

39. A spacecraft in outer space increases its velocity by firing its rockets. How can hot gases escaping from its rocket engine change the velocity of the craft when there is nothing in space for the gases to push against? (9.2)

40. A cue ball travels across a pool table and collides with the stationary eight ball. The two balls have equal masses. After the collision, the cue ball is at rest. What must be true regarding the speed of the eight ball? (9.2)

41. Consider a ball falling toward Earth. (9.2)
- **a.** Why is the momentum of the ball not conserved?
- **b.** In what system that includes the falling ball is the momentum conserved?

42. A falling basketball hits the floor. Just before it hits, the momentum is in the downward direction, and after it hits the floor, the momentum is in the upward direction. (9.2)
- **a.** Why isn't the momentum of the basketball conserved even though the bounce is a collision?
- **b.** In what system is the momentum conserved?

43. Only an external force can change the momentum of a system. Explain how the internal force of a car's brakes brings the car to a stop. (9.2)

44. Children's playgrounds often have circular-motion rides. How could a child change the angular momentum of such a ride as it is turning? (9.2)

Applying Concepts

45. Explain the concept of impulse using physical ideas rather than mathematics.

46. Is it possible for an object to obtain a larger impulse from a smaller force than it does from a larger force? Explain.

47. **Foul Ball** You are sitting at a baseball game when a foul ball comes in your direction. You prepare to catch it bare-handed. To catch it safely, should you move your hands toward the ball, hold them still, or move them in the same direction as the moving ball? Explain.

48. A 0.11-g bullet leaves a pistol at 323 m/s, while a similar bullet leaves a rifle at 396 m/s. Explain the difference in exit speeds of the two bullets, assuming that the forces exerted on the bullets by the expanding gases have the same magnitude.

49. An object initially at rest experiences the impulses described by the graph in **Figure 9-15.** Describe the object's motion after impulses A, B, and C.

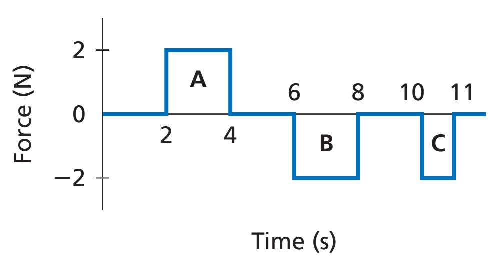


Figure 9-15

50. During a space walk, the tether connecting an astronaut to the spaceship breaks. Using a gas pistol, the astronaut manages to get back to the ship. Use the language of the impulse-momentum theorem and a diagram to explain why this method was effective.

51. Tennis Ball As a tennis ball bounces off a wall, its momentum is reversed. Explain this action in terms of the law of conservation of momentum. Define the system and draw a diagram as a part of your explanation.

52. Imagine that you command spaceship *Zeldon*, which is moving through interplanetary space at high speed. How could you slow your ship by applying the law of conservation of momentum?

53. Two trucks that appear to be identical collide on an icy road. One was originally at rest. The trucks are stuck together and move at more than half the original speed of the moving truck. What can you conclude about the contents of the two trucks?

54. Explain, in terms of impulse and momentum, why it is advisable to place the butt of a rifle against your shoulder when first learning to shoot.

55. Bullets Two bullets of equal mass are shot at equal speeds at blocks of wood on a smooth ice rink. One bullet, made of rubber, bounces off of the wood. The other bullet, made of aluminum, burrows into the wood. In which case does the block of wood move faster? Explain.

Mastering Problems

9.1 Impulse and Momentum

56. Golf Rocío strikes a 0.058-kg golf ball with a force of 272 N and gives it a velocity of 62.0 m/s. How long was Rocío's club in contact with the ball?

57. A 0.145-kg baseball is pitched at 42 m/s. The batter hits it horizontally to the pitcher at 58 m/s.
a. Find the change in momentum of the ball.
b. If the ball and bat are in contact for 4.6×10^{-4} s, what is the average force during contact?

58. Bowling A force of 186 N acts on a 7.3-kg bowling ball for 0.40 s. What is the bowling ball's change in momentum? What is its change in velocity?

59. A 5500-kg freight truck accelerates from 4.2 m/s to 7.8 m/s in 15.0 s by the application of a constant force.
a. What change in momentum occurs?
b. How large of a force is exerted?

60. In a ballistics test at the police department, Officer Rios fires a 6.0-g bullet at 350 m/s into a container that stops it in 1.8 ms. What is the average force that stops the bullet?

61. Volleyball A 0.24-kg volleyball approaches Tina with a velocity of 3.8 m/s. Tina bumps the ball, giving it a speed of 2.4 m/s but in the opposite direction. What average force did she apply if the interaction time between her hands and the ball was 0.025 s?

62. Hockey A hockey player makes a slap shot, exerting a constant force of 30.0 N on the hockey puck for 0.16 s. What is the magnitude of the impulse given to the puck?

63. Skateboarding Your brother's mass is 35.6 kg, and he has a 1.3-kg skateboard. What is the combined momentum of your brother and his skateboard if they are moving at 9.50 m/s?

64. A hockey puck has a mass of 0.115 kg and is at rest. A hockey player makes a shot, exerting a constant force of 30.0 N on the puck for 0.16 s. With what speed does it head toward the goal?

65. Before a collision, a 25-kg object was moving at +12 m/s. Find the impulse that acted on the object if, after the collision, it moved at the following velocities.
a. +8.0 m/s
b. −8.0 m/s

66. A 0.150-kg ball, moving in the positive direction at 12 m/s, is acted on by the impulse shown in the graph in **Figure 9-16.** What is the ball's speed at 4.0 s?

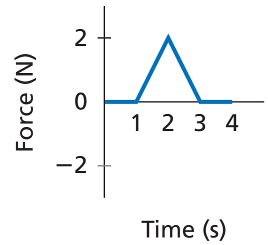


Figure 9-16

67. Baseball A 0.145-kg baseball is moving at 35 m/s when it is caught by a player.
a. Find the change in momentum of the ball.
b. If the ball is caught with the mitt held in a stationary position so that the ball stops in 0.050 s, what is the average force exerted on the ball?
c. If, instead, the mitt is moving backward so that the ball takes 0.500 s to stop, what is the average force exerted by the mitt on the ball?

68. **Hockey** A hockey puck has a mass of 0.115 kg and strikes the pole of the net at 37 m/s. It bounces off in the opposite direction at 25 m/s, as shown in **Figure 9-17.**

a. What is the impulse on the puck?

b. If the collision takes 5.0×10^{-4} s, what is the average force on the puck?

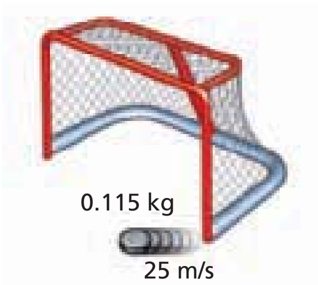


Figure 9-17

69. A nitrogen molecule with a mass of 4.7×10^{-26} kg, moving at 550 m/s, strikes the wall of a container and bounces back at the same speed.

a. What is the impulse the molecule delivers to the wall?

b. If there are 1.5×10^{23} collisions each second, what is the average force on the wall?

70. **Rockets** Small rockets are used to make tiny adjustments in the speeds of satellites. One such rocket has a thrust of 35 N. If it is fired to change the velocity of a 72,000-kg spacecraft by 63 cm/s, how long should it be fired?

71. An animal rescue plane flying due east at 36.0 m/s drops a bale of hay from an altitude of 60.0 m, as shown in **Figure 9-18.** If the bale of hay weighs 175 N, what is the momentum of the bale the moment before it strikes the ground? Give both magnitude and direction.

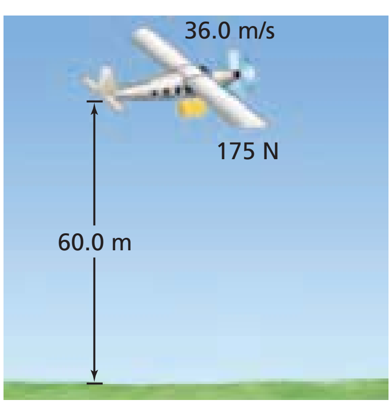


Figure 9-18

72. **Accident** A car moving at 10.0 m/s crashes into a barrier and stops in 0.050 s. There is a 20.0-kg child in the car. Assume that the child's velocity is changed by the same amount as that of the car, and in the same time period.

a. What is the impulse needed to stop the child?

b. What is the average force on the child?

c. What is the approximate mass of an object whose weight equals the force in part b?

d. Could you lift such a weight with your arm?

e. Why is it advisable to use a proper restraining seat rather than hold a child on your lap?

9.2 Conservation of Momentum

73. **Football** A 95-kg fullback, running at 8.2 m/s, collides in midair with a 128-kg defensive tackle moving in the opposite direction. Both players end up with zero speed.

a. Identify the "before" and "after" situations and draw a diagram of both.

b. What was the fullback's momentum before the collision?

c. What was the change in the fullback's momentum?

d. What was the change in the defensive tackle's momentum?

e. What was the defensive tackle's original momentum?

f. How fast was the defensive tackle moving originally?

74. Marble C, with mass 5.0 g, moves at a speed of 20.0 cm/s. It collides with a second marble, D, with mass 10.0 g, moving at 10.0 cm/s in the same direction. After the collision, marble C continues with a speed of 8.0 cm/s in the same direction.

a. Sketch the situation and identify the system. Identify the "before" and "after" situations and set up a coordinate system.

b. Calculate the marbles' momenta before the collision.

c. Calculate the momentum of marble C after the collision.

d. Calculate the momentum of marble D after the collision.

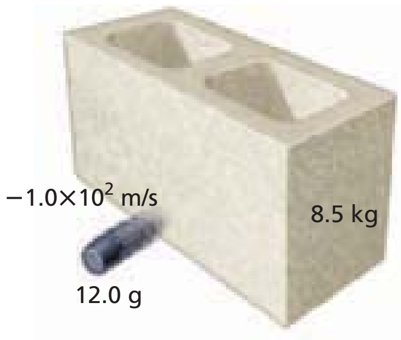
e. What is the speed of marble D after the collision?

75. Two lab carts are pushed together with a spring mechanism compressed between them. Upon release, the 5.0-kg cart repels one way with a velocity of 0.12 m/s, while the 2.0-kg cart goes in the opposite direction. What is the velocity of the 2.0-kg cart?

 For more problems, go to Additional Problems, Appendix B.

76. A 50.0-g projectile is launched with a horizontal velocity of 647 m/s from a 4.65-kg launcher moving in the same direction at 2.00 m/s. What is the launcher's velocity after the launch?

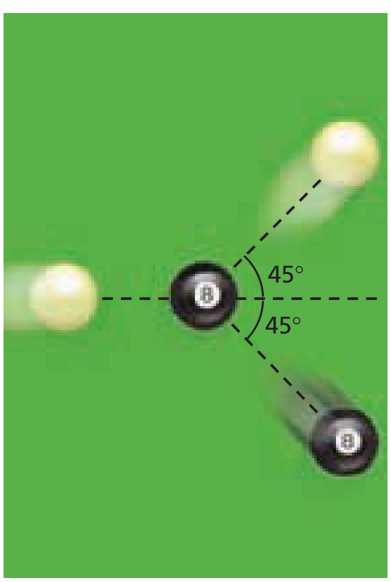
77. A 12.0-g rubber bullet travels at a velocity of 150 m/s, hits a stationary 8.5-kg concrete block resting on a frictionless surface, and ricochets in the opposite direction with a velocity of -1.0×10^2 m/s, as shown in **Figure 9-19.** How fast will the concrete block be moving?



■ **Figure 9-19**

78. **Skateboarding** Kofi, with mass 42.00 kg, is riding a skateboard with a mass of 2.00 kg and traveling at 1.20 m/s. Kofi jumps off and the skateboard stops dead in its tracks. In what direction and with what velocity did he jump?

79. **Billiards** A cue ball, with mass 0.16 kg, rolling at 4.0 m/s, hits a stationary eight ball of similar mass. If the cue ball travels 45° above its original path and the eight ball travels 45° below the horizontal, as shown in **Figure 9-20,** what is the velocity of each ball after the collision?



■ **Figure 9-20**

80. A 2575-kg van runs into the back of an 825-kg compact car at rest. They move off together at 8.5 m/s. Assuming that the friction with the road is negligible, calculate the initial speed of the van.

81. **In-line Skating** Diego and Keshia are on in-line skates and stand face-to-face, then push each other away with their hands. Diego has a mass of 90.0 kg and Keshia has a mass of 60.0 kg.

a. Sketch the event, identifying the "before" and "after" situations, and set up a coordinate axis.

b. Find the ratio of the skaters' velocities just after their hands lose contact.

c. Which skater has the greater speed?

d. Which skater pushed harder?

82. A 0.200-kg plastic ball moves with a velocity of 0.30 m/s. It collides with a second plastic ball of mass 0.100 kg, which is moving along the same line at a speed of 0.10 m/s. After the collision, both balls continue moving in the same, original direction. The speed of the 0.100-kg ball is 0.26 m/s. What is the new velocity of the 0.200-kg ball?

Mixed Review

83. A constant force of 6.00 N acts on a 3.00-kg object for 10.0 s. What are the changes in the object's momentum and velocity?

84. The velocity of a 625-kg car is changed from 10.0 m/s to 44.0 m/s in 68.0 s by an external, constant force.

a. What is the resulting change in momentum of the car?

b. What is the magnitude of the force?

85. **Dragster** An 845-kg dragster accelerates on a race track from rest to 100.0 km/h in 0.90 s.

a. What is the change in momentum of the dragster?

b. What is the average force exerted on the dragster?

c. What exerts that force?

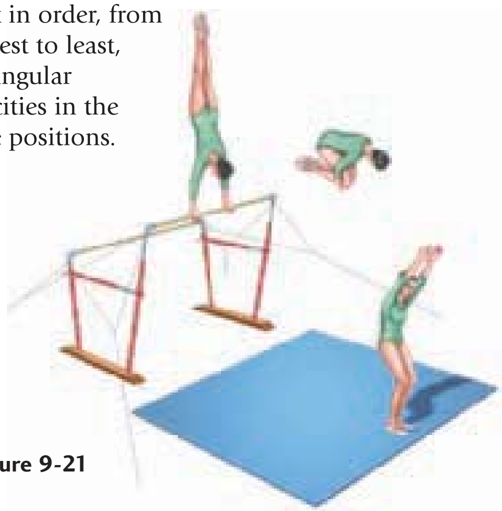
86. **Ice Hockey** A 0.115-kg hockey puck, moving at 35.0 m/s, strikes a 0.365-kg jacket that is thrown onto the ice by a fan of a certain hockey team. The puck and jacket slide off together. Find their velocity.

87. A 50.0-kg woman, riding on a 10.0-kg cart, is moving east at 5.0 m/s. The woman jumps off the front of the cart and lands on the ground at 7.0 m/s eastward, relative to the ground.

a. Sketch the "before" and "after" situations and assign a coordinate axis to them.

b. Find the cart's velocity after the woman jumps off.

88. **Gymnastics** **Figure 9-21** shows a gymnast performing a routine. First, she does giant swings on the high bar, holding her body straight and pivoting around her hands. Then, she lets go of the high bar and grabs her knees with her hands in the tuck position. Finally, she straightens up and lands on her feet.
   a. In the second and final parts of the gymnast's routine, around what axis does she spin?
   b. Rank in order, from greatest to least, her moments of inertia for the three positions.
   c. Rank in order, from greatest to least, her angular velocities in the three positions.



■ **Figure 9-21**

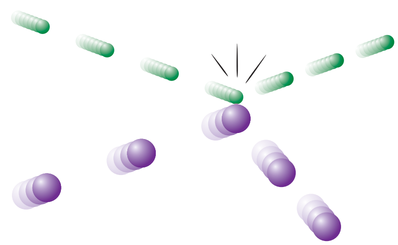
89. A 60.0-kg male dancer leaps 0.32 m high.
   a. With what momentum does he reach the ground?
   b. What impulse is needed to stop the dancer?
   c. As the dancer lands, his knees bend, lengthening the stopping time to 0.050 s. Find the average force exerted on the dancer's body.
   d. Compare the stopping force with his weight.

Thinking Critically

90. **Apply Concepts** A 92-kg fullback, running at 5.0 m/s, attempts to dive directly across the goal line for a touchdown. Just as he reaches the line, he is met head-on in midair by two 75-kg linebackers, both moving in the direction opposite the fullback. One is moving at 2.0 m/s and the other at 4.0 m/s. They all become entangled as one mass.
   a. Sketch the event, identifying the "before" and "after" situations.
   b. What is the velocity of the football players after the collision?
   c. Does the fullback score a touchdown?

91. **Analyze and Conclude** A student, holding a bicycle wheel with its axis vertical, sits on a stool that can rotate without friction. She uses her hand to get the wheel spinning. Would you expect the student and stool to turn? If so, in which direction? Explain.

92. **Analyze and Conclude** Two balls during a collision are shown in **Figure 9-22,** which is drawn to scale. The balls enter from the left, collide, and then bounce away. The heavier ball, at the bottom of the diagram, has a mass of 0.600 kg, and the other has a mass of 0.400 kg. Using a vector diagram, determine whether momentum is conserved in this collision. Explain any difference in the momentum of the system before and after the collision.



■ **Figure 9-22**

Writing in Physics

93. How can highway barriers be designed to be more effective in saving people's lives? Research this issue and describe how impulse and change in momentum can be used to analyze barrier designs.

94. While air bags save many lives, they also have caused injuries and even death. Research the arguments and responses of automobile makers to this statement. Determine whether the problems involve impulse and momentum or other issues.

Cumulative Review

95. A 0.72-kg ball is swung vertically from a 0.60-m string in uniform circular motion at a speed of 3.3 m/s. What is the tension in the cord at the top of the ball's motion? (Chapter 6)

96. You wish to launch a satellite that will remain above the same spot on Earth's surface. This means the satellite must have a period of exactly one day. Calculate the radius of the circular orbit this satellite must have. *Hint: The Moon also circles Earth and both the Moon and the satellite will obey Kepler's third law. The Moon is* 3.9×10^8 *m from Earth and its period is 27.33 days.* (Chapter 7)

97. A rope is wrapped around a drum that is 0.600 m in diameter. A machine pulls with a constant 40.0 N force for a total of 2.00 s. In that time, 5.00 m of rope is unwound. Find α, ω at 2.00 s, and I. (Chapter 8)

 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. When a star that is much larger than the Sun nears the end of its lifetime, it begins to collapse, but continues to rotate. Which of the following describes the conditions of the collapsing star's moment of inertia (I), angular momentum (L), and angular velocity (ω)?

- (A) I increases, L stays constant, ω decreases
- (B) I decreases, L stays constant, ω increases
- (C) I increases, L increases, ω increases
- (D) I increases, L increases, ω stays constant

2. A 40.0-kg ice-skater glides with a speed of 2.0 m/s toward a 10.0-kg sled at rest on the ice. The ice-skater reaches the sled and holds on to it. The ice-skater and the sled then continue sliding in the same direction in which the ice-skater was originally skating. What is the speed of the ice-skater and the sled after they collide?

- (A) 0.4 m/s
- (B) 0.8 m/s
- (C) 1.6 m/s
- (D) 3.2 m/s

3. A bicyclist applies the brakes and slows the motion of the wheels. The angular momentum of each wheel then decreases from 7.0 kg·m^2/s to 3.5 kg·m^2/s over a period of 5.0 s. What is the angular impulse on each wheel?

- (A) -0.7 kg·m^2/s
- (B) -1.4 kg·m^2/s
- (C) -2.1 kg·m^2/s
- (D) -3.5 kg·m^2/s

4. A 45.0-kg ice-skater stands at rest on the ice. A friend tosses the skater a 5.0-kg ball. The skater and the ball then move backwards across the ice with a speed of 0.50 m/s. What was the speed of the ball at the moment just before the skater caught it?

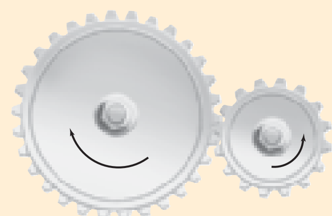
- (A) 2.5 m/s
- (B) 3.0 m/s
- (C) 4.0 m/s
- (D) 5.0 m/s

5. What is the difference in momentum between a 50.0-kg runner moving at a speed of 3.00 m/s and a 3.00×10^3-kg truck moving at a speed of only 1.00 m/s?

- (A) 1275 kg·m/s
- (B) 2550 kg·m/s
- (C) 2850 kg·m/s
- (D) 2950 kg·m/s

6. When the large gear in the diagram rotates, it turns the small gear in the opposite direction at the same linear speed. The larger gear has twice the radius and four times the mass of the smaller gear. What is the angular momentum of the larger gear as a function of the angular momentum of the smaller gear? *Hint: The moment of inertia for a disk is $\frac{1}{2}mr^2$, where m is mass and r is the radius of the disk.*

- (A) $-2L_{small}$
- (B) $-4L_{small}$
- (C) $-8L_{small}$
- (D) $-16L_{small}$



7. A force of 16 N exerted against a rock with an impulse of 0.8 kg·m/s causes the rock to fly off the ground with a speed of 4.0 m/s. What is the mass of the rock?

- (A) 0.2 kg
- (B) 0.8 kg
- (C) 1.6 kg
- (D) 4.0 kg

Extended Answer

8. A 12.0-kg rock falls to the ground. What is the impulse on the rock if its velocity at the moment it strikes the ground is 20.0 m/s?

✓ Test-Taking TIP

If It Looks Too Good To Be True

Beware of answer choices in multiple-choice questions that seem ready-made and obvious. Remember that only one answer choice for each question is correct. The rest are made up by test-makers to distract you. This means that they might look very appealing. Check each answer choice carefully before making your final selection.